The MATH Olympian

A novel by

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"Courage, sacrifice, determination, commitment, toughness, heart, talent, guts. That's what little girls are made of; the heck with sugar and spice."

– Bethany Hamilton, professional surfer

Contents

Prologue

"I'm ready."

My voice is barely audible. I'm terrified.

Mom wraps me in a tight hug, and we cling to each other, shielding our bodies from the howling wind that's typical for a mid-March morning in Nova Scotia. I glance at my watch.

8:49 a.m.

I see a man stare as he walks past us, puzzled by the sight of a broad seventeen-year-old towering over her mother. He, of course, has no idea what's racing through my mind at this moment.

As Mom and I squeeze each other one final time, she knows that I don't need another pep talk or any more words of encouragement. We gently step away from each other, and Mom gets back in the car. Before she closes the door, she turns to give me one last look: a forced, nervous smile.

"I'm ready," I say, as calmly as possible, trying to convince myself that the words I've just spoken are indeed true.

Mom drives off, and I find myself alone, standing a few feet from the main entrance of the Cape Breton regional school board. I take a deep breath, to slow my pounding heart.

8:50 a.m.

In ten minutes, I'll be writing the Canadian Mathematical Olympiad, joining forty-nine other high school students from across the country who also qualified to write Canada's toughest math contest.

One exam. Three hours. Five questions.

I'm the last person to qualify, the student closest to the cut-off. But today, that doesn't matter. Whether I'm ranked first or fiftieth, I'm in. And that means I have a chance.

The chance to achieve my childhood dream.

8:51 a.m.

I walk up the steps to the school board entrance. As I close the door behind me, I come face-to-face with a slim lady with long black hair, who greets me with a look of intensity. She looks just like Gillian Lowell, but thirty years older.

I flinch and take a step back.

"Bethany MacDonald," says the woman, staring into my eyes. "All of Cape Breton is rooting for you today."

I nod, at a loss for words.

An older well-dressed man comes to the rescue. He introduces himself as Mr. MacKay, the school board superintendent. He asks me to follow him towards the conference room, a large open space he has reserved this morning just for me.

I walk into a room with multiple plaques and pictures hanging on the side walls, with an oval-shaped mahogany desk set right in the centre. I take the seat farthest from the door, where I can see the big clock by looking straight up.

"So, Bethany, how tall are you?"

"Six feet," I reply, knowing that this is the easiest question I'll be asked all morning.

"And yet you're not a basketball player?"

"No. I'm a runner."

"I know," says Mr. MacKay. "You're the captain of the cross-country team at Sydney High School."

I raise my eyebrow. Seeing my reaction, the superintendent smiles.

"Bethany, from what I've heard, you've been breaking stereotypes your whole life."

Mr. MacKay says he'll give me some time to get ready. As the door closes, I'm left all alone, with just my thoughts to keep me company as I prepare myself for the challenge ahead.

8:54 a.m.

The International Mathematical Olympiad (IMO) is the world championship of problem-solving for high school students. Nearly one hundred countries are invited to this year's IMO, with each country sending their top six teenage "mathletes".

Ever since my twelfth birthday, I've wanted to be a Math Olympian.

That improbable hope, that one day I'd wear the red and white and represent my country, has sustained me over the past six years. But now I'm in Grade 12, heading to university in the fall, and this is my last chance.

Will I make it? In three hours, I'll know the answer.

Albert, Raju, and Grace are guaranteed to make the team; they're light years beyond the rest of us. Albert Suzuki represented Canada at the IMO the past two years, winning a gold medal both times. Raju Gupta went to the IMO last year, and is a shoo-in again this year. Grace Wong just missed Team Canada by one spot last time, and no one will deny her from making the top six this year.

Because of the four-hour time difference between Nova Scotia and British Columbia, I know that Grace is still sleeping. As the clock ticks ominously above me, I think about my closest friend and reflect on how far we've come since that summer day in Vancouver, when we made our pact.

Since that evening, I've trained non-stop, over thirty hours a week for nearly two years, while juggling all of my responsibilities at school. I've sacrificed so much to make it this far.

And yet, I know the odds are stacked against me.

8:55 a.m.

The Canadian IMO team is determined by a secret formula unknown to the fifty of us writing today's competition. All we know is that each math contest is assigned a certain weight, with this final Olympiad exam being the most important. Our scores from all the contests are then added together, from which the top six will be decided.

I'm frustrated and angry at what happened during the previous contests, where my test anxiety flared up at the worst possible time. I know I am much, much better than how I've performed, and this is my last chance to prove it.

Because of how far back I am from the current top six, my only hope is "The Rule": that the winner of today's Canadian Mathematical Olympiad automatically gets a spot on the IMO team.

Even though this is by far the hardest contest we'll write all year, Albert is sure to get a perfect score, just as he did the year before. So I need fifty out of fifty myself, and tie Albert for the top score in Canada. That's the only way I'll make it.

I need to write five complete solutions in just three hours, wowing the judges with an elegant and flawless performance, just like a figure skater at the Olympic trials.

My figure skating analogy triggers a thought – a bad thought – and I wince.

I don't need to be thinking about *that*. Especially not right now.

8:57 a.m.

I'm reminded of Gillian Lowell yet again, and I recall the spiteful words she said to me four months ago.

"You risked everything on a stupid dream, trying to be an Olympian in math. In math!"

She spoke loudly enough to be heard by everyone else.

"Bethany, get a life."

I crack a smile, finally realizing that Gillian was right all along. I did get a life.

A life more fulfilling than anything I could have ever imagined.

The door opens. The superintendent walks in, and glances at the clock.

8:58 a.m.

"Bethany, shall we begin?"

"Yes," I reply, sitting up straight with my back firmly against the chair.

Mr. MacKay walks over and places in front of me a stack of plain white paper, three blue pens, and five sealed envelopes numbered #1 through #5. He asks me if I have any questions, and I shake my head.

"Okay. As soon as it's nine o'clock, you can start."

Before walking out the door, he pauses and looks back. "Good luck, Bethany. We're all so proud of you."

I stare at the clock above me, and my eyes follow the red second hand moving quickly to the top.

8:59 a.m.

At that instant, I know there's an 84.5° angle formed by the hour hand and the minute hand. Recalling the memory of that special day in Halifax, and everything that's happened since, I feel a sense of peace.

I'm ready.

I watch the red second hand make another clockwise rotation until it once again points directly north.

9:00 a.m.

I rip open the folders, and spend a few minutes studying the five questions.

These problems look hard. Really hard. But this is the Canadian Mathematical Olympiad. Of course they're hard.

As Grace reminded me, most university math professors couldn't solve even one of these five problems. But to be fair, those math professors haven't spent *three thousand hours* training for a moment like this.

I close my eyes and think about the life-changing decision I made on my twelfth birthday, and how this one decision led me to experience hundreds of ups and downs and twists and turns over the past six years.

It's been an amazing ride. In just three hours, this roller-coaster journey will come to an end.

And now I get to write the ending to this story. To my story.

I open my eyes and begin.

The Canadian Mathematical Olympiad

Problem #1

Determine the value of:

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{3/1000}}{9^{3/1000}+3}+\cdots+\frac{9^{998/1000}}{9^{998/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}$$

Problem #2

Find all real solutions to the following system of equations.

$$\begin{cases}\dfrac{4x^2}{1+4x^2}=y\\[2ex]\dfrac{4y^2}{1+4y^2}=z\\[2ex]\dfrac{4z^2}{1+4z^2}=x\end{cases}$$

Problem #3

Twenty-five men sit around a circular table. Every hour there is a vote, and each must respond *yes* or *no*. Each man behaves as follows: on the n^{th} vote, if his response is the same as the response of at least one of the two people he sits between, then he will respond the same way on the $(n+1)^{\text{th}}$ vote as on the n^{th} vote; but if his response is different from that of both his neighbours on the n^{th} vote, then his response on the $(n+1)^{\text{th}}$ vote will be different from his response on the n^{th} vote. Prove that, however everybody responded on the first vote, there will be a time after which nobody's response will ever change.

Problem #4

Let ΔABC be an isosceles triangle with $AB = AC$. Suppose that the angle bisector of $\angle B$ meets AC at D and that $BC = BD + AD$. Determine $\angle A$.

Problem #5

Let m be a positive integer. Define the sequence $x_0, x_1, x_2, \ldots$ by $x_0 = 0$, $x_1 = m$, and $x_{n+1} = m^2 x_n - x_{n-1}$ for $n = 1, 2, 3 \ldots$. Prove that an ordered pair (a, b) of non-negative integers, with $a \le b$, gives a solution to the equation $\frac{a^2+b^2}{ab+1} = m^2$ if and only if (a, b) is of the form (x_n, x_{n+1}) for some $n \ge 0$.

The Canadian Mathematical Olympiad, Problem #1

Determine the value of:

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{3/1000}}{9^{3/1000}+3}+\cdots+\frac{9^{998/1000}}{9^{998/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}$$

Problem #1: Sum of Exponents

I stare at the first problem, not sure where to start.

I circle the first term in the expression of Problem #1, the one with the ugly exponent $9^{1/1000}$. Am I actually supposed to calculate the 1000th root of 9? Without a calculator, I know that's not possible.

There has to be an insight somewhere. This is an Olympiad problem, and all Olympiad problems have nice solutions that require imagination and creativity, rather than a calculator.

I re-read the question yet again, and confirm that I have to determine the following sum:

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{3/1000}}{9^{3/1000}+3}+\cdots+\frac{9^{998/1000}}{9^{998/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}$$

There are 999 terms in the sum, and each term is of the form $\frac{9^x}{9^x+3}$. In the first term, x equals $\frac{1}{1000}$; in the second term, x equals $\frac{2}{1000}$; in the third term, x equals $\frac{3}{1000}$; and so on, all the way up to the last term, where x equals $\frac{999}{1000}$.

In the entire expression, there's only one doable calculation, the term right in the middle. I know I can calculate $\frac{9^{500/1000}}{9^{500/1000}+3}$, using the fact that $\frac{500}{1000}=\frac{1}{2}$.

Since raising a quantity to the exponent $\frac{1}{2}$ is the same as taking its square root, I see that

$$\frac{9^{500/1000}}{9^{500/1000}+3}=\frac{9^{1/2}}{9^{1/2}+3}=\frac{\sqrt{9}}{\sqrt{9}+3}=\frac{3}{3+3}=\frac{3}{6}=\frac{1}{2}$$

But other than this, I'm not sure what to do. Twirling my pen and closing my eyes, I concentrate, hoping for a spark.

One idea comes to mind: setting up a "telescoping series". My mentor, Mr. Collins, introduced me to this beautiful technique years ago at one of our Saturday afternoon sessions at Le Bistro Café. Before explaining the concept to me, Mr. Collins first gave me a simple question of adding five fractions:

Without using a calculator, determine $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}$

I solved Mr. Collins' problem by finding the common denominator. In this case, the common denominator is 60, the smallest number that evenly divides into each of 2, 6, 12, 20, and 30. So the answer is

$$\frac{30}{60}+\frac{10}{60}+\frac{5}{60}+\frac{3}{60}+\frac{2}{60}=\frac{30+10+5+3+2}{60}=\frac{50}{60}=\frac{5}{6}$$

And then I remembered Mr. Collins' smile as he gave me another addition problem:

Determine $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}$

This time, it took me almost fifteen minutes to get the answer. Most of the time was spent trying to figure out the common denominator, which I eventually determined to be 2520. But it was a tedious process of checking and re-checking all of my calculations.

After Mr. Collins congratulated me on getting the right answer, he pointed to the nine fractions on my sheet of paper and asked if there was a pattern. After staring at the numbers for a while, I saw it:

2 = 1 × 2	**6** = 2 × 3	**12** = 3 × 4
20 = 4 × 5	**30** = 5 × 6	**42** = 6 × 7
56 = 7 × 8	**72** = 8 × 9	**90** = 9 × 10

Mr. Collins suggested I write $\frac{1}{90}$ as the difference of two fractions: $\frac{1}{90}=\frac{1}{9}-\frac{1}{10}$. He then asked whether there were any other terms in this expression that could also be written as the difference of two fractions. I eventually saw that $\frac{1}{2}=\frac{1}{1}-\frac{1}{2}$ and $\frac{1}{6}=\frac{1}{2}-\frac{1}{3}$.

Once I saw the pattern, I discovered this amazing solution, called a "telescoping series":

$\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}$ can be re-written as

$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{9}\right)+\left(\frac{1}{9}-\frac{1}{10}\right)$$

This is just $\frac{1}{1}-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}+\frac{1}{7}-\frac{1}{8}+\frac{1}{8}-\frac{1}{9}+\frac{1}{9}-\frac{1}{10}$.

Since one negative fraction cancels a positive fraction with the same value, all the terms in the middle get eliminated:

$$\frac{1}{1}\cancel{-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\frac{1}{4}-\frac{1}{5}+\frac{1}{5}-\frac{1}{6}+\frac{1}{6}-\frac{1}{7}+\frac{1}{7}-\frac{1}{8}+\frac{1}{8}-\frac{1}{9}+\frac{1}{9}}-\frac{1}{10}$$

Like a giant telescope that collapses down to a small part at the top and a small part at the bottom, this series collapses to the difference $\frac{1}{1}-\frac{1}{10}$, which equals $\frac{9}{10}$. So the answer is $\frac{9}{10}$.

That day, Mr. Collins showed me several problems where the answer can be found using a telescoping series, where a seemingly-tedious calculation can be solved with elegance and beauty.

The key is to represent each term as a difference of the form $x-y$, where y is called the "subtrahend" and x is called the "minuend". From Mr. Collins' examples, I learned that the series telescopes every time the subtrahend of one term equals the minuend of the following term.

As I recall that lesson with Mr. Collins many years ago, I'm hopeful that I can use this technique to solve the first problem of the Canadian Math Olympiad. I look at Problem #1 again, reminding myself of what I need to determine.

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{3/1000}}{9^{3/1000}+3}+\cdots+\frac{9^{998/1000}}{9^{998/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}$$

I start with the general expression $\frac{9^x}{9^x+3}$ and try to write it down as the difference of two functions, so that the subtrahend of each term equals the minuend of the following term.

I try a bunch of different combinations to get the difference to work out to $\frac{9^x}{9^x+3}$ such as the expression $\frac{1}{3^x}-\frac{1}{3^x+1}$ which almost works but not quite. I

attempt other combinations using every algebraic method I know. All of a sudden, I realize the futility of my approach.

The denominator doesn't factor nicely, so this approach cannot work. Oh no.

9:19 a.m.

I feel the first bead of sweat on my forehead, and wonder if I'm going to get another "math contest anxiety attack". I close my eyes and take a deep breath, knowing that if I start to panic and lose focus, my chances of becoming a Math Olympian are over.

Calm down, Bethany, calm down. There's lots of time left. You can do this.

I think about the soothing words of Mr. Collins, and am reminded of another important problem-solving strategy I learned from him: simplify the problem by breaking it into smaller and easier parts, in order to find a pattern.

I can do that.

I don't want to deal with the horrible expression given in the problem, a complicated sum of nearly one thousand fractions. I've seen enough contest problems to know that the number 1000 is a distracter, and that it has nothing to do with the question. By making the number big, the problem looks a lot more intimidating than it actually is.

For example, in the addition question that Mr. Collins posed to me that day, as soon as I realize that the series telescopes, it doesn't matter whether there are nine fractions or nine thousand fractions. In the former the answer is $\frac{1}{1}-\frac{1}{10}=\frac{9}{10}$ and in the latter the answer is $\frac{1}{1}-\frac{1}{9001}=\frac{9000}{9001}$. The final answer is different, but at its heart, it's the exact same problem.

I'm sure the same is true with this Olympiad problem. Especially being the first question, I know there has to be a short and elegant solution. Remembering the advice of Mr. Collins, I decide to simplify the problem in order to discover a pattern, which will then allow me to solve the actual problem.

I change the denominator from 1000 to 4, to have just a few terms to play with. So now, instead of the exponents ranging from $\frac{1}{1000}$ to $\frac{999}{1000}$, I only have to consider $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$.

Instead of adding 999 ugly terms as in the actual problem, I only have three terms in the simplified problem. By making the expression easier, I am hopeful that I'll discover something interesting.

So my simplified problem is to determine the value of

$$\frac{9^{1/4}}{9^{1/4}+3}+\frac{9^{2/4}}{9^{2/4}+3}+\frac{9^{3/4}}{9^{3/4}+3}$$

This looks much more reasonable. The middle expression is easy: I figured this out ten minutes earlier.

$$\frac{9^{2/4}}{9^{2/4}+3}=\frac{9^{1/2}}{9^{1/2}+3}=\frac{\sqrt{9}}{\sqrt{9}+3}=\frac{3}{3+3}=\frac{3}{6}=\frac{1}{2}$$

As I ponder how to calculate the values of $\frac{9^{1/4}}{9^{1/4}+3}$ and $\frac{9^{3/4}}{9^{3/4}+3}$, a few ideas occur to me. I scribble some calculations on my notepad, add up the two fractions, and am surprised that the sum is exactly one.

$$\frac{9^{1/4}}{9^{1/4}+3}+\frac{9^{3/4}}{9^{3/4}+3}=1$$

Interestingly, the first and last terms of my simplified problem add up to 1. I have a hunch that this might also be true in the more complicated Olympiad problem with 999 terms.

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{3/1000}}{9^{3/1000}+3}+\cdots+\frac{9^{998/1000}}{9^{998/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}$$

To my delight, the hunch is correct.

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}=1$$

I run through the calculations one more time, double-checking that I haven't made any mistakes.

Yes, the terms in the numerator perfectly match the terms in the denominator, and the sum is indeed one.

I'm confident that this pattern continues, and am not surprised to discover that

$$\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{998/1000}}{9^{998/1000}+3}=1$$

I suddenly feel a lump in my throat. I know how to solve the Olympiad problem.

All I need to do is to apply the technique I discovered in Mrs. Ridley's class seven years ago, when I was in Grade 5. I can't believe it.

It's the Staircase.

1

"Two-hundred ten!" I blurted out.

Every head in the classroom turned towards me. Several students stared at me in shock. Mrs. Ridley stood there with her back leaning against the chalkboard and her jaw dropped, and for several uncomfortable seconds that seemed like an eternity, I could hear the sound of my own heart thumping.

One person broke the silence. Of course, it was Gillian.

"Is that the answer?"

My Grade 5 teacher turned towards me and smiled. "Yes, it is."

Michael, the loud boy sitting to my left, began clapping.

"It looks like Gillian finally has some competition."

A few people joined in the applause, which prompted Gillian to turn around from her seat right in the middle of the second row and glare at Michael, who was sitting directly behind her.

"Bethany," said Mrs. Ridley. "Well done. How did you do that so quickly?"

I shrugged and looked at my shoes.

"Let me try that again," pressed Mrs. Ridley, taking a couple of steps towards me. "You couldn't have just added up the numbers. Can you show all of us how you got the answer?"

I slouched back into my chair, and stared at my notepad with the picture of the two joining staircases.

Mrs. Ridley knew that public speaking terrified me. Surely she knew that every time I spoke, my baritone voice reduced half the class to giggles.

"I'm sorry," she said. "I shouldn't have asked you to share in front of the entire class. But how did you add up the numbers from one to twenty so quickly? Can you show me what you did, Bethany?"

I shook my head and closed my notepad, afraid that I'd get in trouble if Mrs. Ridley saw all the pictures I had drawn in class that day.

I couldn't tell Mrs. Ridley that I was tired from the mindless drills we did every day, and that I needed something to do to pass the time. I couldn't explain to my teacher that I was only drawing staircases because math was so boring.

Even though my head was down, I could feel everyone staring at me.

All of a sudden, I heard a high-pitched cackle from Vanessa, the freckled redhead sitting in front of me. As always, I was sure it was her best friend Gillian who leaned over and whispered something cruel.

Gillian and Vanessa were inseparable, and formed a tight clique with Alice and Amy, the two Chinese twins who sat directly in front of them in the first row. Whenever a teacher wasn't around, the four of them called me names like Big Ugly Bethany. I was so much bigger and taller, yet they were the ones bullying me.

When Mrs. Ridley posed her usual "mental math" question at the halfway mark of the class, it was a crazy coincidence that my doodling provided the spark needed to answer her question of calculating the sum of the first twenty positive integers: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20.

Or maybe the creative spark came from all those jigsaw puzzles I'd been doing since I was five, and realizing how the two staircases would fit perfectly together.

Mrs. Ridley's mental math questions made me uncomfortable, and I hated how Gillian always made it a personal competition. Even though I understood ideas like fractions and long division, I just wasn't fast at doing calculations. I could do them – they were easy – it just took time.

Gillian's brain worked faster than the rest of ours, and she always got the mental math question first.

Until now.

"Bethany," Mrs. Ridley whispered. "Instead of sharing your solution in front of the class, perhaps you could write it down for me, and I'll present it to the class? Would you do this for me?"

I looked at the clock, and saw that class wouldn't finish for another thirty minutes.

I wanted to get away to a place where I could be safe – where I could be alone.

But unlike Meg, the main character in the Madeleine L'Engle book I finished last night, I couldn't do a "wrinkle in time". I was stuck here, in three dimensions, with no chance of escape.

Sighing, I stared at Mrs. Ridley and nodded. I picked up my pen and opened my notepad.

Mrs. Ridley returned to the front of the classroom and started explaining another drill while I concentrated on writing my "solution".

I wondered whether I should talk about the staircase. If I explained the staircase, would the truth come out that I drew the picture only because I was bored and was just scribbling on my page? Could I demonstrate my solution without the two staircases?

No, I couldn't. I had to draw the staircases.

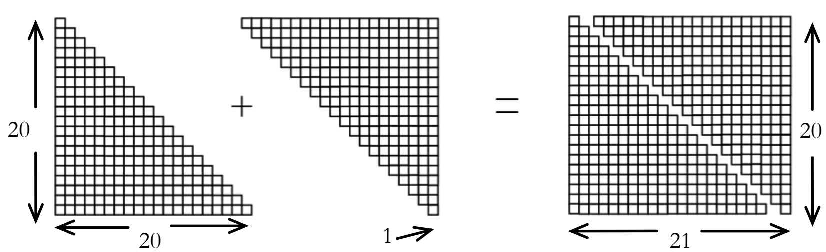
I ripped out a fresh sheet of paper, and took out a small ruler from my pencil case.

I began writing. About fifteen minutes later, when I was sure everything was just right, I slowly lifted my hand. Mrs. Ridley saw me, walked over and took my sheet of paper.

With the class working on something else, Mrs. Ridley began reading what I had written:

Answer = One staircase

Two staircases joined together = One rectangle



Two staircases	= 21 × 20
One staircase	= 21 × 10
Answer	= 210

I didn't take a breath as I stared at Mrs. Ridley reading over my solution. Her eyes kept moving up and down the page. Finally she glanced up with a confused look on her face.

"Sorry, I don't get it."

My teacher didn't understand. My shoulders sagged. Mrs. Ridley bent down and leaned so close that I could smell the perfume on her face.

"Bethany, can you explain it to me?"

We whispered back and forth, pointing to the diagram, until it all clicked in her mind.

"Oh, wow," said Mrs. Ridley. "I understand. Yes, I understand."

Mrs. Ridley got the attention of the class, and walked up to the front of the classroom, placing my sheet of paper on top of the fancy projector before turning the machine on. After a minute, the projector was ready and my staircase diagram filled the screen at the front of the classroom.

"Class, listen carefully," said Mrs. Ridley. "In the mental math question, I asked you to calculate the sum 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20. I want to show you how Bethany solved the problem."

"Why doesn't Bethany explain it?" interrupted Gillian. "I'm sure she'd be happy to."

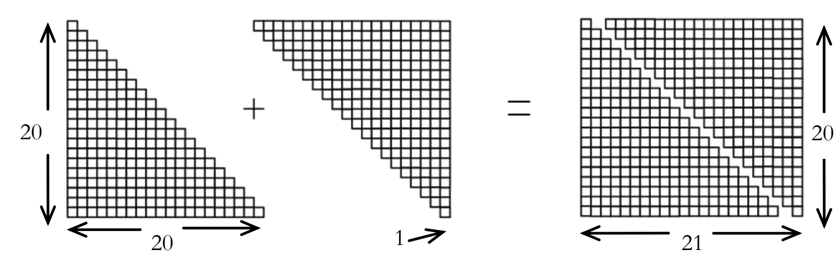
Vanessa started giggling. Mrs. Ridley shook her head and pointed to the diagram.

"As I was saying, here is Bethany's solution. Look at the staircase on the left. There's one square in the top row, two squares in the second row, three squares in the third row, all the way down to twenty squares in the last row. So the answer to the mental math question is equal to the number of squares in that *one* staircase."

Mrs. Ridley paused. "Do you all understand Bethany's picture?"

A bunch of heads nodded. I held my breath.

"Now here is Bethany's amazing idea. Make a copy of the staircase and *flip* it," said Mrs. Ridley, pointing to the second staircase.



She then pointed to the rectangle on the right.

"And if we take those two staircases and stitch them together, we end up with a rectangle that's twenty squares high and twenty-one squares wide."

She explained how each row in the rectangle would have 21 squares, since the first row has 1+20 = 21 squares, the second row has 2+19 = 21 squares, and the third row has 3+18 = 21 squares, and so on until we get to the twentieth and final row, which has 20+1 = 21 squares.

"Oh, I get it!" exclaimed Michael. "You take two staircase-shaped pieces and join them together – and you get a rectangle. It's like a jigsaw puzzle."

"That's exactly right, Michael. Now, who can explain the rest of Bethany's solution?"

Gillian raised her hand. With her chin lifted up, she spoke in her usual high-pitched tone.

"The answer is the number of squares in one staircase. Twice the answer is the number of squares in two staircases, which is the same as the number of squares in that big rectangle. The number of squares in that big rectangle is twenty-one times twenty, width times height. So the number of squares in *two* staircases is twenty-one times twenty."

"Good," said Mrs. Ridley. "So how many squares are in one staircase?"

"Half of that," said Gillian. "Twenty-one times ten."

"Thank you, Gillian. As always, that's an excellent explanation. Did everyone understand that?"

One student shook his head to indicate that he hadn't. Mrs. Ridley called on Gillian again to explain why the answer had to be half the number of squares in that big 21×20 rectangle, and why this worked out to 21×10=210. This time, the student nodded in understanding.

"Well done, Gillian," said Mrs. Ridley. "And well done, Bethany."

I smiled.

Mrs. Ridley glanced towards the middle of the classroom, beaming at the two of us.

"We should call you the Bethany and Gillian team. The B & G Team."

My smile disappeared.

Gillian raised her hand. "The G should come first. The G & B Team."

Mrs. Ridley ignored the comment and turned to write something on the chalkboard.

After a few seconds, Gillian snickered and looked at Vanessa. She pointed to herself with her index finger and then pointed her thumb directly at me. She lowered her voice.

"G & B. Good and Bad."

Upon hearing Vanessa's cackle, Mrs. Ridley turned back to face us. "What was that?"

"Oh, nothing," replied Vanessa. "I was just laughing about something that happened yesterday."

Mrs. Ridley shook her head. After a slight hesitation, she picked up her chalk and continued writing on the board. Gillian looked at Vanessa again. She pointed to herself and then stuck her finger towards me.

"G & B. Girl and Boy."

This time Vanessa's laugh could be heard by the entire classroom. Mrs. Ridley glared hard at Vanessa.

"I'm sorry, Mrs. Ridley," replied Vanessa. "It won't happen again."

I glanced at the clock. The bell would ring in five minutes. I looked at the clock in desperation.

Hurry up, clock! Move faster!

I saw Gillian take out a sheet of paper and two pencils. As she began sketching, I could see her smile getting broader every minute.

I leaned forward to see the image Gillian was creating.

A feeling of nausea came over me. It was a picture of two people. The girl on the left had a big *G* on her shirt. She had long flowing dark hair, tanned skin, and a perfectly-shaped face. The girl on the right had a big *B* on her shirt. She was really tall, with broad shoulders, a pointed nose, frazzled hair, and huge feet. The girl on the left looked like a ballerina, while the girl on the right looked like a deformed tree.

I didn't need to be reminded which one was me. A tear crept out of my eye.

Gillian scribbled some words below the two pictures and showed her masterpiece to Vanessa and the twins sitting in front of them. They burst into laughter.

As Gillian turned the page towards Vanessa, I saw the word *BIGFOOT* written below my portrait, and felt more tears flowing down my cheeks.

Then I saw the eight-letter word *GORJEOUS* written under Gillian's portrait.

Mrs. Ridley angrily walked over and grabbed the picture. When she looked at what Gillian had created, her face flushed.

The bell rang. As people began to pack up their stuff, Mrs. Ridley held up her hand.

"Gillian, Vanessa, Alice, Amy, Bethany. You're not dismissed."

After everyone had left, there were just six of us in the classroom: Mrs. Ridley, me, and Gillian's gang. The twins looked scared knowing that they were in big trouble, while Gillian and Vanessa looked indifferent. I quickly wiped the tears away from my eyes, and sniffled.

"Gillian Lowell," snapped Mrs. Ridley, pointing to her picture. "Did you draw this?"

"Yes," she said, nonchalantly. "But it was just a joke. I didn't mean it."

Mrs. Ridley glared at her. "Gillian, what you did was absolutely unacceptable. It was deeply hurtful, and you should be ashamed of yourself. What do you have to say to Bethany?"

Gillian turned to me and shrugged. "Sorry."

"Apologize like you mean it."

"Sorry," said Gillian, a bit louder but still totally unconvincing.

With a pained expression in her eyes, Mrs. Ridley sighed and looked at me: "Do you accept her apology?"

"Yeah," I replied, and stood up from my seat, taking my books and binder with me.

A strange feeling came over me, just as I was about to exit the classroom. It was a sense of conviction, something I had never felt before.

I grabbed a piece of chalk from the side board and wrote down an eight-letter word in big block letters.

GORGEOUS

Standing up straight, I locked eyes with Gillian, and tapped the chalkboard.

"I might not be gorgeous. But at least I can spell it."

2

ELEVATOR OUT OF SERVICE.

"Not again," I muttered.

I slowly walked up the stairs, carrying my heavy backpack, all the way up to the sixth floor.

Today was June 15. My twelfth birthday. It was a terrible day from start to finish, and the broken elevator was the icing on the cake.

Grade 6 would finish in a couple of weeks. I couldn't wait for school to be over. Then I wouldn't have to see Gillian and Vanessa and anybody else until September, when we'd all start Grade 7 at a new school.

I didn't need to be reminded that there was a party at Gillian's this evening, since June 15 was also Gillian's twelfth birthday.

I couldn't blame people for wanting to go to her place instead of mine. After all, Gillian had the biggest house in Cape Breton, complete with a backyard pool, while Mom and I rented an apartment on the sixth floor of a dumpy old building, where the elevator broke at least once a week.

I slowly opened the door.

"Happy birthday, darling!"

"Thanks, Mom."

"How was school today?"

"Terrible," I said, bursting into tears. Mom turned off the stove and reached out to give me a hug.

I told Mom about what Gillian said to me in the girls' locker room. That I had no friends. That I was the only person in the entire Grade 6 class not invited to her birthday party. That my father left Mom a month before I was born because he knew that the baby would end up being a stupid loser.

Mom sighed and hugged me tighter. She didn't say anything. She didn't need to.

I hated my life.

After a while, I calmed down. I just wanted to be alone, and get back to reading *Harry Potter*.

"Can I turn off the TV?"

"Of course, darling. Dinner will be ready in just a few minutes."

While Mom returned to the kitchen, I looked for the remote control in our tiny living room. "Mom, where's the remote?"

Mom didn't look up from the pot she was stirring.

"I don't know, but the power button's just under the TV. Dinner's almost ready."

Just as I was about to hit the power button, I saw a clip of six teenagers wearing matching red and white shirts, writing math equations on a big chalkboard. They were all smiling and laughing, and the camera focussed on the short freckled girl in the middle, with her straight auburn hair falling just below her shoulders.

A deep male voice resonated from the television set.

"Are you a good problem-solver? Think you can match wits with the best in the country? When we come back, you'll meet Rachel Mullen and the rest of Canada's team to the International Math Olympiad."

I stood there in silence as a commercial came on.

Math Olympiad?

"Ready for dinner, honey?"

I continued to stare at the screen.

"Bethany?"

"Uh, not ready. Two minutes – just want to see something first."

Confused by my sudden interest in the news, Mom walked over and stood behind the worn-down Lazy-Boy recliner and motioned for me to take her seat. I nodded and sat down, finding the remote control sandwiched between the covers of the recliner.

"What are we watching?" asked Mom.

The commercials were still rolling. "I'm not sure."

The screen returned to the CBC news, and the anchor's face appeared. I turned up the volume.

"When Rachel Mullen was growing up in Brandon, Manitoba, she was an over-enthusiastic child interested in everything, especially mathematics. Even though some of her teachers encouraged her to pursue other interests, she stuck with the subject she enjoyed the most. And today, she is one of our country's brightest young minds, representing Canada next month at the biggest stage of them all, the International Mathematical Olympiad. Here is her story."

I stared at the TV as Rachel stood in front of the classroom, pointing at some equations on the whiteboard and explaining something to a teenage boy who nodded in agreement. In the next shot, Rachel was sitting opposite a middle-aged reporter with the Canadian flag in the background. She laughed at something the reporter said, and smiled at him.

"I was a hyperactive kid who had a hard time concentrating on tasks, always moving from one activity to another. I drove my parents crazy. But my parents were patient with me, and they were so supportive of my dreams and ambitions – even though some of my dreams only lasted a few weeks! Luckily, I met an amazing teacher in Grade 7 who noticed my interest in puzzles, and she introduced me to math contests. She saw my potential even before I did, but eventually I realized it too."

"And what was that?" asked the reporter.

"That I could develop a passion for math and grow to love it. And through writing contests, I could get invited to math camps and meet people from all across Canada. Even though we grew up in different cities and provinces, we have so much in common. They'll be my closest friends for the rest of my life."

"What do you like about math, Rachel?"

"The beauty, the patterns. Everyone thinks math is about memorizing formulas and rules – but it's not. At its heart, math is about problem-solving. I'm not the smartest person at my school, but I'm probably the most creative and imaginative. That's how I developed as I trained for the Math Olympiad."

"Tell me about the Olympiad. Are you looking forward to it?"

"Definitely. I've been wanting to make the team for the past three years, and finally got it on my last try. I'm nervous, but I'm ready."

The news story then switched to an older man whose perfectly-parted thin blond hair appeared almost child-like, and looked so strange on top of his head. As the man began to speak, a small caption appeared on the bottom of the screen: "J. William Graham, Executive Director, Canadian Mathematical Society".

"The six members of our Canadian IMO team were selected from among *two hundred thousand* students from Grades 7 to 12 who participated this year in local, provincial, and national mathematics competitions. They represent the very best of Canada, and will be excellent ambassadors for our country at the Math Olympiad."

"Can you tell me more about the Olympiad competition?" asked the reporter.

"I'd be happy to," said Dr. Graham. "Our six team members will pit their skills against the top math students from over one hundred countries. They will attempt to solve six problems over nine hours, a mathematical 'hexathlon' that requires exceptional problem-solving skills, mathematical understanding, daring, and imagination – the types of skills that we Canadians will require if we're going to be at the forefront of innovation in the twenty-first century."

"Excuse me, Dr. Graham. Did you say six problems over *nine* hours?"

"That's right. Six problems over nine hours. Preparing for a competition like this one requires years of training, just like the athletes at the Summer and Winter Olympics. Just as our Canadian athletes amaze us with their physical prowess and push the boundaries of athletic performance, our *mathletes* do the same thing with their intellectual prowess. They are truly an inspiration to Canada."

Rachel's face reappeared in front of the screen. She was sitting next to the Canadian flag.

"Rachel, what advice would you give to a young person who might be watching this?"

She paused. "Find out what gets you excited and passionate. Some of my teachers tried to turn me away from math because I was a girl, and felt I should pursue other hobbies and interests. But I stuck with math because that's what got me excited. That's what inspired my passion. That's what grew my self-esteem and confidence. That's what added meaning and joy to my life. I found my voice, and because I did, I'm now a Math Olympian."

The story ended. I sat there, on the recliner, unable to move.

Mom was saying something to me, but I couldn't hear her.

I had an epiphany of what direction my life could take over the next six years. It was a moment of shocking clarity, a voice from deep within shouting into my head and heart.

I trembled with excitement. Since coming up with the "staircase" insight in Mrs. Ridley's class last year, I had been struck by the beauty of math but

never had any teacher draw that out. I was never challenged or stretched in class, so I found the subject boring and mindless.

But I felt that there was something more to math. There had to be something more to math – for me.

I was moved by Rachel's words – self-esteem, confidence, meaning, joy – and in that one moment, I knew.

"Mom," I whispered, "I want to be a Math Olympian."

"That's nice, darling," said Mom, walking towards the kitchen. "Ready to eat dinner?"

"Mom!" I shouted. "I'm not kidding! I want to be a Math Olympian."

She turned to face me, and stood in front of the recliner. She saw the intensity in my eyes. Her face froze, and for a brief moment I saw her face flush.

"What's wrong, Mom?"

She hesitated.

"Nothing, Bethany. I'm fine."

She reached over and hugged me, squeezing me a lot tighter than usual. She took the remote from my hand, turned off the TV, and led me towards the dining table.

We chit-chatted a bit, but not as much as we normally did. I ate my chicken noodle soup, silently chewing each bite, daydreaming about the future. Mom looked a bit flustered, but assured me everything was okay.

Once we had finished eating, Mom invited me to unwrap my birthday present, a board game called Scrabble.

"You'll love this, darling. It's perfect for you."

"Thanks, Mom. This looks really fun. You want to play?"

"Of course I do. But I have one final surprise for you."

As I was reading the rules for Scrabble, Mom returned to the kitchen, and came back with a chocolate cupcake with a candle in the middle.

"Now make a wish, birthday girl. Anything you want."

I closed my eyes.

My mind kept coming back to a single thought that gave me goose bumps, and renewed me with a sense of joyful hope.

I want to be a Math Olympian.

I smiled and blew out the candle.

3

“Oh my God. Rachel won a gold medal!”

Mom looked up from her book. She stood up from her seat at the dining table and slowly walked towards our computer in the living room.

“Check this out,” I said, giving Mom my chair.

Looking over her shoulder, I re-read the story I just found on *cbc.ca*.

Canadian solves her way to Math Olympic Gold

When Rachel Mullen clears customs at Pearson International Airport this afternoon, she will declare that she is in possession of a bright and shiny object obtained during her recent trip abroad – a gold medal from the International Mathematical Olympiad (IMO).

Ms. Mullen, a seventeen-year old from Brandon, Manitoba, represented Canada at this year’s IMO, the world championship of mathematical problem-solving for high school students. Out of one hundred countries, the Canadian team finished in fifteenth place, with one gold medal and five bronze medals amongst its six team members.

“This year’s IMO was one of the most difficult in years. The problems were extremely challenging yet all six students performed exceptionally well,” said J. William Graham, the executive director of the Canadian Mathematical Society, the organization responsible for the selection and training of Canada’s IMO team.

The annual IMO contest is set by an international jury of mathematicians, with one from each participating country. On each day of the contest, three questions had to be solved within a time limit of four-and-a-half hours.

Six hundred students wrote this year’s IMO. Gold medals were given to the fifty students with the highest total score. Ms. Mullen correctly solved five of the six problems, placing tenth overall.

“I’m thrilled with our team’s result,” said Ms. Mullen, who will head to the University of Waterloo in September on a full scholarship. “The problems were so hard this year! I’m still in shock that I won a gold medal. I will treasure this experience for the rest of my life.”

I felt goose-bumps on my arms. Rachel had done it – the best in Canada, one of the best in the world.

“I want to be a Math Olympian too.”

Mom sighed.

“You said that last month, right after we saw that TV clip. Do you really mean it?”

“Yeah,” I replied instantly. “Totally.”

Mom shook her head. “Trust me, Bethany. You don’t want to pursue this. It’s not worth it.”

“How would you know?” I asked.

“From personal experience,” said Mom, hesitating.

“What personal experience?”

Mom changed the subject. “If you go for this Math Olympiad, you’re going to need to make sacrifices and train for thousands of hours. You won’t have the time to hang out with friends, play on sports teams, join school clubs, and just enjoy being a teenager.”

“But what if this is what I want to do?”

“You don’t, Bethany,” said Mom, rising up from the chair. “Trust me, you don’t.”

“Why?” I pressed.

“Because these young Olympians do nothing else but train. They have no social life because there’s so much pressure to perform. It’s a terrible thing to ask a young person to endure. And I don’t want you to get hurt like that.”

“I want to do this.”

“Remember when we were watching Wimbledon a few weeks ago?”

“You’re changing the subject.”

“No, I’m not. Remember Wimbledon? Think about all those players we were cheering for. Do you know how they all got so good?”

I didn’t answer.

“Their parents enrolled them in private tennis academies when they were little kids. They had personal coaches. They practiced all day, every day. Their parents hired tutors to help them with their school work in the evenings. None of them had a normal life. I want you to have a normal life.”

“But what if I don’t want a normal life?”

"Look at our circumstances, Bethany! I'm raising you on my own. I'm sure all these Math Olympian kids have parents who are math professors, who teach them all sorts of complicated math, and work with them for hours every night. Or their parents have tons of money and can send their kids to schools where their teachers give them special coaching. I can't do that."

I remembered the part in the TV clip when Rachel spoke so lovingly of her parents, who supported her dreams and encouraged her every step of the way.

"Why can't you be supportive of my dream?"

"Because your dream isn't realistic," she responded. "If you put all of your eggs in one basket and that basket breaks, what happens then? You get shattered. Your life gets completely shattered."

Mom's voice trailed off. She looked away from me. A few seconds later, I heard sniffling.

I stared in disbelief. "Mom, are you . . . crying?"

Mom didn't answer. She walked away from the computer and sat on the recliner, dabbing her eyes with her fingers. I had only ever seen Mom cry once before, years ago, when Grandpa got really sick.

I sat on the floor facing Mom, and looked up at her. Neither of us spoke for several minutes.

"I'm sorry," I said, meaning it.

After a long pause, Mom wiped her eyes with another tissue and put her hand on my arm.

"When I was your age," whispered Mom, "I wanted something big. It took over my life. It robbed me of my childhood. It robbed me of everything. I don't want you to have to go through what I did."

"Go through what, Mom?" I asked.

She was so private about her past – about her childhood, about her adult life, even about my father's identity. I knew Mom grew up in Cape Breton but she didn't hang out with people who knew her well; she was an only child so I had no relatives other than Grandpa since Grandma died before I was born. As for my father, all I knew was that he was a star hockey player and would have made it to the NHL had he not gotten injured, and that he left Mom for a woman in Alberta while Mom was pregnant with me.

Mom looked into my eyes. I could sense she was ready to share. I moved closer.

"You know how I used to do figure skating."

I nodded, remembering what my Geography teacher told me in Grade 4, that she watched Mom skate on TV. From my teacher, I learned that Mom won the provincials three years in a row, between the ages of seventeen and nineteen. Until then, I had no idea that Mom was a star athlete.

"I hated figure skating. Even though I was the provincial champion, the pressure got to me. I made myself vulnerable. I don't want you to open yourself like that, and risk so much for a goal that's so uncertain."

I held Mom's hands and didn't say anything. I wondered why she was so upset about being the best figure skater in Nova Scotia.

"Please, darling. Don't risk your future on trying to be amazing at math. It's better to be well-rounded. It's better to have a normal life."

Mom reached over and gave me a hug. But inside I felt empty.

Normal was so boring.

Whenever I read books like *Harry Potter* or *The Hobbit*, I saw people doing stuff that was interesting. They were pursuing adventures. They were having fun. They were living out their dreams.

All I could think of was Rachel and that TV clip, where she talked about the passion she got from striving for the Math Olympiad and how that pursuit gave her life meaning and joy.

"Are you okay?" asked Mom.

"Yeah, I'm fine," I replied, not meaning it.

Mom put her hand on my shoulder. She held it there for a bit, and then went back to her seat at the dining table and continued reading her book. I went to the kitchen to get myself a glass of orange juice, and stood there in silence.

I wanted life to be interesting.

After standing by the fridge for a few minutes, staring blankly into space, I had an idea. Returning to the computer, I did a Google search on "International Math Olympiad".

How hard could these Olympiad problems be? After all, I was pretty good at math already, and knew that I would get a lot better in the next few years. Maybe I could pursue this dream . . . secretly.

I clicked on the first link, and after a few more clicks, was taken to a site containing all the IMO problems since the annual event began in 1959.

I chose a year at random, and downloaded the English-version of the problems from that year.

My jaw dropped when I saw the first problem. There were no numbers anywhere.

Wasn't this supposed to be a math contest?

Question #1

Determine all functions $f: R \rightarrow R$ such that the equality

$$f(\lfloor x \rfloor y) = f(x)\lfloor f(y) \rfloor$$

holds for all $x, y \in R$.

(Here, $\lfloor z \rfloor$ denotes the greatest integer less than or equal to z.)

The three problems on Day 1 seemed like they were written in a foreign language. Scrolling down to the first problem of Day 2, I recognized the word "triangle" but not much else.

Question #4

Let *P* be a point inside triangle *ABC*. The lines *AP*, *BP* and *CP* intersect the circumcircle of triangle *ABC* again at the points *K*, *L* and *M*, respectively. The tangent to the circumcircle at *C* intersects the line *AB* at *S*. Suppose that *SC*=*SP*. Prove that *MK*=*ML*.

What was a "circumcentre"? A "tangent"? And what were all those letters *P*, *K*, *L*, *M* to keep track of?

Glancing at the six problems, and not understanding the meaning of a single one, I realized that the Math Olympiad was so much harder than the calculations and formulas I was used to. I found the page with the solutions to each of these six problems and clicked on the link for Question #4.

There were a bunch of complicated geometrical diagrams accompanying each of the two solutions that were posted, with one mentioning the "Tangent-Chord Theorem", and the other mentioning the "Power-of-a-Point Theorem". As I skimmed the two solutions to try to make sense of them, my head started to hurt.

I closed the web browser and got up from the computer table. Moving to Mom's recliner, I sank down into the seat, in shock.

So much for wanting to be a Math Olympian.

I'd been thinking and dreaming about the Math Olympiad every day for the past month. But I now knew I could never get to that level.

Mom was right. The Math Olympiad was just for super-special people, the natural prodigies with math professors for parents, or gifted teenagers whose parents could afford personal tutors.

The Math Olympiad was for the Rachel Mullens of this world, not for ordinary people like me.

My eyes started to well up.

I heard the phone ring, but couldn't move.

"Hello?" said Mom, picking up the phone.

After a couple of seconds, she spoke again. "Hi, Dad. How are you?"

All of a sudden, she began to cry. It started off slowly but her sniffles turned into heavy sobs. I jolted from my seat and turned to face Mom, wondering what had happened to Grandpa.

I stared at Mom as she continued her call. Her voice was muffled and I could only make out certain words through her tears, but one word was unmistakable.

Cancer.

4

"Bye, Grandpa. See you in a couple of days."

We closed the door to his room at Cape Breton Regional Hospital, and walked towards the elevator in silence.

We had visited Grandpa at least four times a week since mid-July, when we learned that Grandpa only had months to live. Even though Grade 7 would be starting in two days, I wanted to come to the hospital as often as possible, and asked Mom if we could go together every night after dinner.

The elevator went down to the main floor of the hospital. We stepped out and headed towards the exit.

"Is that you, Lucy?"

An old man was walking towards us.

Mom was the first to react. "Mr. Collins, what a surprise."

The man gave Mom a warm hug. "I'm so sorry about your father. I'm here to visit him now."

"Thank you," said Mom. "I know he'll be happy to see you."

"Of course," he replied. "Your father and I go back many years."

After an awkward pause, Mom pointed to me. "Mr. Collins, this is my daughter Bethany."

I shook hands with the man, and stared at the thick grey hair on top of his pear-shaped face.

"It's nice to meet you, Bethany," he said, smiling at me. "My name is Taylor Collins, and I was your mother's math teacher at Sydney High School. Your Grandpa and I grew up on the same street."

Mr. Collins invited us to join him in the hospital cafeteria, and offered to buy us any drink we wanted. Mom resisted, but Mr. Collins insisted we join him.

Mr. Collins bought a coffee for Mom and an orange juice for me. We found an empty table, right by the cafeteria door.

He turned to Mom. "How are you, Lucy?"

"Other than the news about Dad, I'm doing well."

"Are you still with the federal government?"

"Same old, same old," she said, nodding.

He chuckled. "I assume that means you really don't like your job."

"No, I do," she said, correcting herself. "The work is good."

I glanced over at Mom, surprised she lied to her former teacher.

I knew Mom hated her job at the Canada Revenue Agency. For the past ten years, she was the administrative assistant to one of the senior directors. The director yelled all the time, especially at Mom. Whenever Mom talked to anyone on the phone, she complained about her boss.

Mr. Collins asked about me. I told him about my boring, ordinary life. He seemed to disagree, and was happy to hear about my favourite books, and my evening Scrabble games with Mom. Unlike Gillian and her friends, Mr. Collins didn't tease me for having a manly voice, and he instantly made me feel comfortable.

"In a couple of days, you'll start Grade 7 at Pinecrest Junior High. Are you excited?"

"Yes. But I'm also nervous."

"That's completely natural, especially when you start at a new school. In a few years, you'll be a student at Sydney High School, where I've been a teacher for the past thirty-nine years. I've had the privilege of working with thousands of talented students throughout my career."

Mom looked at me sheepishly. "He wasn't including me in that list."

Mr. Collins nodded. "Well, you were preoccupied with something else when you were in high school."

Mom forced a smile, and returned to sipping her coffee.

I stood up. "Just need to use the washroom. Be right back."

The ladies' room was just around the corner. As I walked back towards the cafeteria, I could hear Mom and Mr. Collins in a serious conversation.

I stood close to the cafeteria door, where I could hear everything without being seen.

"My granddaughter Ella is really sad. She loves figure skating and wants to compete in the Olympics someday. But her coach got a new job in Toronto and moved out west last month. You know, I feel very strange asking you this, but I was wondering if you'd think about . . ."

"No chance, Mr. Collins."

"But there's no one in Cape Breton more qualified to coach Ella. We would pay whatever you wanted."

"I'm sorry. I'm not interested. After what happened, I have no desire to ever go back to the rink."

"You're right, Lucy. I apologize for asking. Please forgive me for being so insensitive, especially at a time like this."

"It's okay. It's just that I have so many regrets. Because of figure skating, I couldn't go to college or university. And now, years later, I'm stuck in a job I hate."

"Can you start over?"

"It's much too late for that, especially when you're thirty-four years old and only have a high school diploma. Besides, the salary is too good and it's too much of a risk to walk away from a permanent government job, especially because I'm raising Bethany on my own."

"I admire you, Lucy. You've had to deal with so much."

I nervously stepped into the cafeteria and sat back down.

"Is everything okay?" asked Mom.

"Yeah," I replied, patting my stomach. "It must have been something I ate."

Even though Mom looked concerned, I noticed Mr. Collins grinning.

"Bethany," said Mr. Collins. "I can tell you have the ability to think quickly on your feet. From what I've heard, you're an excellent student."

"How do you know?" I asked.

"Cape Breton is a small place, and it turns out that Mrs. Ridley and I are close friends. She told me about your creativity and writing skills, and she thinks the world of you."

"Thank you," I said.

Mr. Collins smiled and sipped his coffee.

"Bethany, from what Mrs. Ridley has told me, I don't think you'll find Grade 7 math either challenging or interesting. The subject is so beautiful, but the curriculum is so boring. Yes, I'll admit it. The curriculum is *boring*.

"Imagine an art class where you spend the entire year practicing brushstrokes and learning how colours combine without actually creating a single painting, or being inspired by how simple techniques could produce an artistic masterpiece revealing elegance and beauty? In art class, you create art and get inspired by beautiful art. But in math class, it's a different story. It's shameful."

"But aren't you changing that?" asked Mom. "I saw your name in the paper about some provincial committee you're on."

"That's right," said Mr. Collins. "I'm chairing the committee that will create the new mathematics curriculum for all of Atlantic Canada. Even though I'm two years from retirement, the next two years will be the most exhausting of my life. But it's a challenge I'm eagerly anticipating."

"Ever since I've known you, you've been involved in a lot of things."

"Yes," he said. "Life is far too busy right now, and I can't possibly take on any more than I've already got on my plate. But it will be worth it when students like Bethany experience the new curriculum."

"When will that happen?" I asked.

"In two years," he replied. "When you start Grade 9."

Mom turned to Mr. Collins.

"In the meantime, what should Bethany do? For the past few years, she's been coming back from school, unexcited. She says school homework is too easy and I can tell she's bored. I'm not confident that Grade 7 will be any different. Is there anything you could recommend for her?"

"Yes," said Mr. Collins, turning to me. "I think you would enjoy writing math contests. Have you ever seen a math contest?"

I nodded, remembering the day we learned about Grandpa.

"I saw a math contest on the internet. Last month."

"What was it called?" he asked.

"The International Mathematical Olympiad."

"Wow, you're ambitious," he said, with a big smile stretched across his face. "You're already solving the problems from the IMO?"

"Oh no," I said, turning red. "I didn't understand the meaning of a single question."

"Well, you're only heading into Grade 7, so that's understandable. From what I've heard about you, I think you have the potential to do well in contests. There's a fun contest open to all Grade 7 students across Canada, and Pinecrest students always participate. Maybe you'll consider participating yourself."

"Definitely," I said.

"How would Bethany get ready for a math contest?" asked Mom.

"Well, it seems like she would already do quite well," replied Mr. Collins. "But if she wanted to excel and get a really high score, she would have to study on her own. Sadly, there aren't any after-school math enrichment programs offered in Cape Breton, unlike bigger cities where students have access to more resources."

Mom tapped her fingers on the table. She looked like she was deep in thought, like at the end of a Scrabble game when the score was really close.

After some chit-chat about the weather, Mom turned to Mr. Collins. "Can I ask you a question?"

"Of course, Lucy."

"Of all the coffee shops you know around town, what's the best place to study?"

"Le Bistro on Charlotte Street," said Mr. Collins. "It's well-lit, the tables are big, and the background music is nice and soft."

Mom took a deep breath.

"That would be perfect. Especially because Le Bistro is so close to the rink."

Mr. Collins looked at Mom in surprise.

"Please tell your granddaughter I will pick her up at her house at one forty-five on Saturday, and will drive her to Centre 200. I will drop her off at Le Bistro around three o'clock, just after she's finished her first session with me, her new figure skating coach."

"Really, Lucy?" said Mr. Collins, beaming. "I'm thrilled to hear this. How much should we pay you?"

"Zero," replied Mom. "I will coach Ella every Saturday. For free."

"I can't tell you how much this means to me. Thank you."

"You're welcome," she said. "Ella and I will go to Le Bistro next Saturday, to meet you – and Bethany."

Mom smiled and continued. "I will drop off Ella at three o'clock, just after Bethany has finished her first session with you, her new math coach."

I stared at Mom in shock.

She looked at Mr. Collins. "After all, there's no one in Cape Breton more qualified to coach Bethany."

Mr. Collins laughed out loud. "That settles it. I accept."

"Mom, you don't have to," I said. "If you don't want to coach . . ."

"It's okay," replied Mom, putting her hand on my shoulder. "I love figure skating. I've always wanted to be a coach."

"So, what do you say, Bethany?" asked Mr. Collins. "Do we have a deal?"

I nodded, at a loss for words.

"Great," he replied. "Well, I should get going and say hello to your Grandpa. It was nice to meet you, Bethany. I'll see you next Saturday."

He turned to Mom, and gave her a hug. "Thank you so much, Lucy."

"No," she whispered. "Thank you."

I watched Mr. Collins walk out of the cafeteria and head towards the elevator. I turned to Mom and saw she had a big smile on her face.

"Thanks, Mom. You're the best."

5

"Next stop, Charlotte Street."

I sighed in relief, and hit the yellow button next to my seat. As the bus turned onto downtown Sydney's main waterfront street, I could see my destination on my right.

As soon as the door opened, I got out and sprinted towards Le Bistro Café, aware that I was already ten minutes late for my first session with Mr. Collins.

I ran into the café, and was grateful to see Mr. Collins sitting by the big table in the corner, waving at me.

"I'm so sorry I'm late," I said, walking towards him. "The bus . . ."

"No problem," said Mr. Collins, interrupting me with a smile that immediately put me at ease.

"I know that the buses here don't always come on time. Let's just end ten minutes later than we planned, okay?"

"Yes," I replied, taking off my backpack and jacket. "Thank you."

"Here, let's get something to drink."

We walked to the counter and lined up behind several people. Looking up at the elaborate menu of artisan sandwiches and specialty drinks that were handwritten using coloured chalk, I realized how different Le Bistro was from the two places Mom and I most often went: McDonald's and Tim Hortons.

While we were waiting to place our order, Mr. Collins asked me about my first week at Pinecrest. I excitedly told him that I had made two new friends, Bonnie and Breanna. We met during homeroom on the first day at our new school, and we had an instant connection since we were all brunettes, and were the three tallest girls in our grade.

I didn't bother telling Mr. Collins that Gillian and her friends were in all of my classes too.

After we sat down with our drinks, a medium coffee for Mr. Collins and a small orange juice for me, Mr. Collins got down to business.

"Bethany, I'm delighted to work with you. Every Saturday afternoon, we'll spend an hour learning together, here at Le Bistro. We'll learn the

heart of mathematics, a beautiful subject that revolves around deep patterns connected together in unexpected ways."

He was moving his hands, waving them in all directions.

"You'll discover how mathematics develops creativity and problem-solving skills, and makes important connections to everything you see in the world. However, you won't be learning any formulas that you'll need to memorize, nor will you be doing a single drill or calculation."

I nodded, pretending to understand, but confused because I thought math was all about formulas, drills, and calculations.

"Are you ready to do some math, Bethany?"

"Yes," I said, picking up my pen.

Mr. Collins placed a stack of white index cards in the centre of the table, and gave the top card to me. "All right, let's get started. On that card, please spell out your first name."

I did what I was told. B-E-T-H-A-N-Y.

"Now, spell it backwards."

I raised my eyebrow, confused by the simple instructions. Staring at my card, I wrote the seven letters of my name in reverse order, one line below: Y-N-A-H-T-E-B.

"Bethany, from your reaction I can tell you're wondering why I'm asking you to do something so basic. So let's make it more interesting. Put down your pen and card, and spell your last name backwards. And this time spell it out loud."

I hesitated. I said my last name a few times out loud, visualizing it in my head. *MacDonald, MacDonald, MacDonald.*

I quickly got the first two letters: "D-L."

MacDonald, MacDonald. Then I got the next letter. "A."

MacDonald, MacDonald. Then I got the next letter. "N."

Repeating the process, I finally came to the end. That was a lot harder than I expected.

"Great work," said Mr. Collins. "Now Bethany, I'd like to introduce to you a problem-solving technique that you can apply to almost any situation in life. The technique is to *simplify the problem by breaking it into smaller parts*.

"Let's take our example with backwards spelling. Instead of thinking of your last name as MacDonald, think of it as Mac – Don – Ald. Three blocks, three letters."

Mr. Collins repeated the three syllables, gesturing with three fingers on his left hand. "Now, Bethany, reverse-spell your last name again. But this time, do it block by block, rather than letter by letter."

Mac – Don – Ald, I thought to myself while staring at my hand, one finger for each syllable.

"D-L-A – N-O-D – C-A-M."

I got that in just a few seconds.

"Great! Now spell *Cape Breton Nova Scotia*, doing exactly what you just did."

Cape – Bre – Ton – Nova – Sco – Tia, I said silently, using both hands to mark the six blocks. I shook my fingers, three times on each hand.

"A-I-T – O-C-S – A-V-O-N – N-O-T – E-R-B – E-P-A-C."

"Excellent. You reverse-spelled twenty letters in less than thirty seconds. See how much simpler that is?"

I nodded.

Mr. Collins gave me more words and phrases to reverse-spell. Each time, I was getting faster and faster.

"Excellent," said Mr. Collins, after about fifteen minutes or so. "Now let's move on to something else. I'm going to write down a ten-letter word and you'll tell me what's special about it."

BOOKKEEPER

"That's another word for librarian?"

"Not quite," said Mr. Collins. "But that's a good guess. A bookkeeper is someone who records business transactions, so it's a synonym of 'accountant'. Perhaps some of your Mom's colleagues at the Canada Revenue Agency are bookkeepers. Do you notice anything unique and special about the spelling of this word?"

"Yes," I replied. "The double letters. O-O then K-K then E-E."

"Very good. It turns out that BOOKKEEPER and SWEETTOOTH are the only two words in the English language that have three consecutive sets of double letters. There aren't any others."

"How about BOOKKEEPERS?" I asked. "And BOOKKEEPING?"

"Good point," said Mr. Collins, nodding. After a short pause, he spoke again.

"Here's a challenge for you, Bethany. Determine a well-known seven-letter word that has two consecutive sets of double letters, where the third and fourth letters are both L."

__ __ L L __ __ __

I looked at my math coach, confused that we had now moved from backwards spelling to word puzzles. He nodded silently, clearly indicating he wanted me to focus on his question.

I quickly saw one word that fit the pattern, BALLBOY. But that didn't have two consecutive sets of double letters. I racked my mind to find other seven-letter words that fit that pattern. After a couple minutes of concentration, I came up with TALLEST, which made me think of myself. And moments after that, I came up with BULLIES, which made me think of Gillian and her friends.

But none of these words had two consecutive sets of double letters.

I looked up at Mr. Collins and shrugged. "I don't know."

"No problem. Have you found any seven-letter words that fit the pattern?"

"Just two," I said, hesitating. "BALLBOY and TALLEST."

"Good. There are a few other words that fit the pattern, like CELLARS and CALLING and WILLING, and also my last name, COLLINS. Now you know you're looking for a seven-letter word with two consecutive pairs of double letters. We talked about a powerful problem-solving strategy earlier. What was that strategy?"

"Simplify the problem."

"That's right. How do we simplify the problem?"

"Break it into small parts."

"Excellent. Let's apply that strategy to this problem. How do we do that?"

"The double letters."

"Right. You want to focus on the double letters. You've got one pair of double letters already, and you know there must be another pair of double letters somewhere else. So what's the next step?"

"Figure out where those double letters can go."

I took three index cards from the stack, one for each of the three possible cases. For each case, I scribbled an "x" where the double letters could go.

x x L L _ _ _   _ _ L L **x x** _   _ _ L L _ **x x**

Mr. Collins smiled. "You made one small mistake. Remember the statement of the problem? What are we looking for? A seven-letter word with what?"

"Two pairs of double letters."

"Two *consecutive* pairs of double letters."

"Oops," I said, flipping over the third index card. So we were down to two cases.

x x L L _ _ _   _ _ L L **x x** _

"Excellent," said Mr. Collins. "Now let's simplify the problem further by breaking it down into even smaller parts. What are the possibilities for these double letters? Look at your first index card, the one where the first two letters are the same. Can the first two letters be a pair of consonants?"

I thought for a few seconds. No, we couldn't have words beginning with BBLL, CCLL, DDLL, FFLL, and so on. That was impossible. The first pair of letters had to be vowels.

Clearly the word couldn't begin with AALL, IILL, or UULL. I thought for a couple of moments and realized that OOLL didn't make any sense either. I racked my brain for seven-letter words beginning with EELL. I smiled.

"Is it EELLIKE?"

"Is it what?"

"You know, EEL-LIKE, like an eel?"

Mr. Collins grinned. "That's a great try, but no, that's not a word."

I turned over the first card. I was convinced that the correct seven-letter word couldn't begin with two consecutive pairs of double letters, since EELLIKE was the only possibility that fit the pattern. I picked up the second index card, the one where I had marked the pattern _ _ L L **x x** _.

I realized that this pair of double letters couldn't be consonants either, since no word would fit a pattern like _ _ L L B B _ or _ _ L L C C _. So this pair of letters had to be vowels.

I was sure that the pattern had to be _ _ L L E E _ or _ _ L L O O _. The other three possibilities didn't make sense.

What could the word be? I figured that the answer would probably be two separate words stuck together to form a single word, like BALL-BOY or MALL-RAT. That made the most sense. But what could the first four letters be? There were plenty of options: BALL, TALL, SELL, FILL, ROLL, PULL, and so on. For each four-letter option, I tried to join it to a three-letter word to form the seven-letter answer.

But what could the last three letters be? EEL was the only possibility that I could think of. What other three-letter words began with EE or OO? BALL-EEL certainly wasn't a word. And the double O's didn't make much sense. Of course, BALL-OOF, BALL-OOM, and BALL-OON weren't words.

And then I saw it.

"BALLOON!"

Several customers turned to stare at me. I put my head down, embarrassed at the attention. But inside I was ecstatic.

Mr. Collins chuckled. "Outstanding. How do you feel?"

I let out a deep breath. "Good. Tired, but good."

Mr. Collins proceeded to give me more word puzzles with missing information. The problems got progressively harder, with the last one requiring me to determine a well-known word with U-F-A in consecutive letters. After fifteen minutes, I finally got it.

"MANUFACTURE!"

"Yes, that's right! Now quick, give me *four* more English words that also have those same three letters appearing consecutively."

"Four more?" I asked. "It took me forever to come up with just one."

"I know you can do it, Bethany. Another powerful strategy is to solve a problem by reducing it to a previously-solved problem. Remember BOOKKEEPERS?"

I laughed, understanding what he was implying. After a slight pause, I came up with four more words that fit the pattern: MANUFACTURES, MANUFACTURING, MANUFACTURED, and MANUFACTURER.

"Well done."

Mr. Collins glanced at his watch. He and I were both surprised that our hour was nearly complete.

"Well, Bethany, it was a pleasure working with you. You're extremely bright, and once you learn a problem-solving strategy, you quickly figure out how to apply it. You must be feeling tired."

I nodded. Although I was drained mentally, I felt alive and exhilarated, a feeling I hadn't experienced since that day in Mrs. Ridley's Grade 5 class nearly eighteen months before.

"Now I want to ask you an important question. How much math do you think we did today?"

Just as I was about to answer, I saw a quirky smile on Mr. Collins' face, as if he were asking me a trick question.

"A little?" I responded. "Simplifying questions into smaller parts. That's math, right?"

"Absolutely. Our context was word puzzles and backwards spelling, but we spent every minute of our time thinking mathematically. The exercises we did are improving your concentration and deductive reasoning skills, and training your mind to make complex associations. After all, math is not about remembering the right formula to get the right answer, but applying your imagination and problem-solving skills to tackle complex problems. Speaking of which, do you use Google?"

"Yes," I said. "All the time."

"Did you know that Google's search algorithm was created by a pair of twenty-three-year-old students in California? Their solution is an ingenious application of Linear Algebra, a field of mathematics you'll first encounter before you graduate from high school. While the Google algorithm is simple, the ability to come up with that solution required extraordinary

creativity and innovative thinking. But these are skills that students can develop. These are the skills I want to develop in you."

"Those Google guys must be rich."

"Indeed. Their breakthrough turned them both into billionaires. It still astounds me that you can rank trillions of websites in the correct order in less than a quarter of a second. But that's what you can do with Linear Algebra, and that's how internet search engines work. By building your skills in critical thinking and deductive reasoning, you will become a more confident, creative problem-solver, able to apply your mind to serve society – just as those Google guys did."

"How can I build that?"

"Build what? Build a billion-dollar company, or build your problem-solving skills?"

I blushed. "The second one."

"I'd recommend you spend some time each week on your own, *cross-training* your mind, the same way sports athletes cross-train their body to enhance their physical performance: tennis players run to develop their endurance; football players lift weights to build muscle; figure skaters do yoga for more flexibility and range of motion."

"Figure skaters do yoga?"

"Absolutely."

"Did Mom ever do yoga?"

"I don't know," said Mr. Collins. "Remember I was her math teacher, not her skating coach."

I smiled, trying to picture Mom as a teenager in a class with Mr. Collins.

"Can you tell me about Mom's figure skating career?" I asked. "There's so much I don't know."

"Let's not talk about that," replied Mr. Collins, suddenly turning serious. "It's not that there's anything to hide; there's just no sense talking about the past."

"Okay," I replied, disappointed.

"Bethany, as I was saying, you need to cross-train your mind, just as an athlete cross-trains her body. As a young *mathlete*, I'd recommend you spend a minimum of thirty minutes a day doing mental gymnastics to develop your mind. Our local newspaper, the Cape Breton Post, has a page of puzzles every day, which are just perfect. Have you heard of the Cryptoquote, the Jumble, the Sudoku, and the Kenken?"

"Just the Sudoku," I said.

"Well, I'd encourage you to get the local paper so that you can do the other puzzles too. They're easy to learn, and they're great training for the mind. We can discuss them next week if you'd like. Would you like to do that?"

I nodded.

"Grampy!"

I turned and saw a petite blonde running towards us, with Mom trailing a few steps behind her.

Mr. Collins introduced me to Ella, his eight-year-old granddaughter. She turned to Mr. Collins and started raving about her first lesson with her new coach. Mom had a smile on her face.

"I had so much fun today, Grampy! Coach Lucy says I'm really good."

"She's right," said Mom, looking at Mr. Collins. "Ella is a natural athlete, and she covers the rink so gracefully. Her jumping skills are amazing for someone her age."

After I told Mom about my first session with Mr. Collins, Ella turned to her grandfather.

"Do you think my Mummy will want to do yoga with me?"

"Why do you ask?" asked Mr. Collins.

"Because Coach Lucy says that if I do yoga, I'll get better at spins and turns. Coach Lucy says that yoga will help me skate better."

"Really?" said Mr. Collins. "I had no idea there was a connection between yoga and figure skating."

I snickered.

"What's so funny?" asked Mom.

"Nothing," I replied, smiling at Mr. Collins. "Nothing at all."

6

"Orange juice is boring, Bethany. Why not try something different this time?"

"What do you suggest?" I asked, pointing to the beverage menu on the wall, with weird-looking names like Chai Latte, London Fog, and Espresso Macchiato written in bright blue chalk.

"How about the Steaming Hot Chocolate?"

"That would be great. Thank you."

Mr. Collins ordered a specialty coffee for himself and a medium hot chocolate for me, with the total coming out to $6.85. Mr. Collins opened up his wallet and took out a ten-dollar bill. Just as the cashier was pressing the register to get the change, he stopped her.

"Hang on. I've got some change. Here's $12.10," he said, handing over a two-dollar coin and a dime.

The cashier looked at him strangely, and entered the amount. She was surprised that the change came out to exactly $5.25, and wordlessly handed Mr. Collins a five-dollar bill and a quarter.

Turning to me, he smiled. "Useful application of math, eh?"

"Do you always do that?"I asked.

"Yes," he replied. "Instead of getting four extra coins, I got one coin and gave up two, and now my wallet is lighter. I have a simple system that's guaranteed to leave me with at most seven coins in my wallet at any time. Like those people who prefer the status quo, I can't stand *change*."

I groaned at the pun, and took my seat at our usual table. Moments later, a young man came by with our drinks. I took my first sip.

"It's good, eh?" said Mr. Collins. "They make it with real chocolate. No powder or syrup."

"It's amazing," I replied, licking my lips. "Thank you so much."

"So, Bethany, what did you think of today's Jumble?"

From my backpack, I took out the puzzle page from the Cape Breton Post, with my completed answers. The Jumble showed a picture of a well-dressed man yawning with the caption: *Why the school superintendent was always so tired.*

To the left of the picture were six words that needed to be unscrambled, with the underlined letters requiring a final unscrambling to form the correct answer, which was always a visual or verbal pun.

I handed my completed Jumble to Mr. Collins.

SNURB	→	**BU**R**N**S
AAMMD	→	MA**DA**M
OIAPT	→	PA**TI**O
WEFRE	→	**F**EW**ER**
OOOODV	→	V**OO**D**O**O
EEEDSC	→	SE**C**E**DE**

The underlined letters were **BUNDATIFEROOOCDE**, which unscrambled to the answer of "Why the school superintendent was always so tired".

He was **"BORED" OF EDUCATION**

Mr. Collins laughed. "I'll add that to my list of *pun*-ishing jokes."

I noticed a familiar-looking man a few tables over waving to Mr. Collins. He smiled and waved back at the well-dressed man who looked around forty or forty-five.

"Who's that?" I asked.

"He's the mayor of Sydney, and he's also a former student of mine. His youngest niece is in your class at Pinecrest."

"What's her name?"

"Gillian Lowell."

I tried to act as nonchalant as possible. "I see."

Mr. Collins, perhaps sensing my reaction, decided it was time to start working.

"Before I state today's question, let me ask you if Pinecrest's hallway is still the same as when I was last there, with several hundred lockers on one side, where the lockers are numbered #1, #2, #3, and so on?"

"Yup, it's still like that."

"And what locker do you have?"

"Locker #100."

"Oh, how perfect. By the way, how many students are at the school this year?"

"Miss Carvery said that there were about 450," I replied, recalling my vice-principal's message on the first day of school.

"Great. Here's our problem for today. Pretend that there are exactly 450 students at Pinecrest, and they conspire to play a strange game with all the lockers. Suppose all of the lockers are initially closed. The first student, the one with Locker #1, goes down the hallway and opens each of the lockers. After she's done, the second student, the one with Locker #2, goes down the hallway and closes all the lockers that are multiples of 2. After he's done, the third student, the one with Locker #3, changes the state of all lockers that are multiples of 3, closing the open lockers and opening the closed lockers.

"And this process continues, with each student going down the hallway one after the other, altering the lockers whose numbers are multiples of their locker number. You've got Locker #100, so when it's your turn, you would alter the 100^{th}, 200^{th}, 300^{th}, and 400^{th} locker. Does that make sense?"

"I think so," I replied. "So when it's my turn, if Locker #100 is open, I would close it?"

"That's right. And if it's closed, you would open it. You do the same thing when you get to Locker #200, Locker #300, and Locker #400."

"I see."

"So the exercise continues until all 450 students have completed their tour down the hallway. The last student, the one with Locker #450, would just change the state of Locker #450. When she's done, the game ends. Here's my question. At the end, how many lockers are open, and which ones are they?"

Oh boy. There was no way I was going to write down 450 marks on my notepad and figure out one by one which lockers were going to remain open. That would take several hours, at least.

I looked at Mr. Collins and shrugged.

"Since we started working together two months ago, we've used a powerful problem-solving strategy almost every week. What is that strategy?"

"Break down a difficult problem into smaller parts."

"That's right. Last week, you showed that a positive integer is divisible by 99 precisely when it's divisible by both nine and eleven. So in coming up with the rule for divisibility by 99, you split the task into two easier problems, and then put it together to solve the harder problem."

"But I don't see how to do that with this locker question. You can't break it down into smaller parts."

"You're right," he replied. "But you can do something else. What's the scariest part of this problem?"

"The number 450. It's too hard to keep track of that many people."

"If it were a smaller number, would the problem be less scary?"

"Of course," I replied. "But you can't just pretend you've got less people to make the problem easier."

"*Fewer* people, not *less* people. Why can't you pretend you've got fewer people in the problem?"

I stared at him. "Well, you're not allowed. You said there were 450 people and 450 lockers. You can't just change the number of people to something less . . . I mean, fewer."

Mr. Collins grinned. "The great thing about problem-solving is that we can change whatever we feel like, and remove any restriction that we think is placed upon us. By pretending than we have fewer than 450 people, we're simplifying the problem so that we can discover a pattern. Once we figure out what the pattern is, we can see if it holds for the actual problem with 450 students."

"Are we really allowed to do that?"

"Definitely," said Mr. Collins. "My suggestion is for you to simplify the locker problem by pretending that there are only a handful of students at Pinecrest, say exactly twenty-five students. By considering this small scenario, you'll be able to figure out what's going on, enabling you to solve the actual problem."

Mr. Collins handed me a fresh sheet of paper.

I created a square grid, and filled in the numbers from one to twenty-five on the rows and on the columns. I then marked the first three rows of my diagram.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**
1	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
2		C		C		C		C		C		C		C		C		C		C		C		C	
3			C			O			C			O			C			O			C			O	

"That's really good, Bethany. Before you go any further, can you explain what you are doing?"

"Sure," I said. "The rows represent the students, and the columns represent the lockers. The first student opens all the lockers; that's why I marked the entire first row with the letter O. The second student closes every other locker; that's why in the second row, I marked all the even lockers with the letter C. And for the third student, for each multiple of three, every O locker turns into a C, and vice-versa."

"That's an excellent explanation. Now please fill out the rest."

It took just a few minutes to fill out the rest of the grid, especially since every student after the thirteenth person could only alter one locker. I highlighted the lockers that remained open at the end, and recognized the pattern instantly.

"So which lockers remain open?"

"Locker #1, #4, #9, #16, and #25," I said, surprised.

"Do you notice anything interesting about these five numbers?"

"They're all perfect squares," I replied. "1×1, 2×2, 3×3, 4×4, and 5×5. Cool."

I put down my pen, certain that this pattern held for the more difficult problem.

"For 450 lockers, the answer will be all the perfect squares less than 450. We're done."

"Not so fast, Bethany," said Mr. Collins, raising up his hand like a referee. "You've taken a small sample and used it to find a pattern. This nice pattern suggests that the answer is all the perfect squares less than 450, but doesn't actually prove it. You need to add a step in between: first find the pattern, then clearly explain the reason for the pattern, and then use your explanation to determine the answer to the actual problem with 450 students."

"How do I do that?"

"You tell me. Why do we get this nice pattern?"

I thought about it for a few minutes, looking at the rows to see if I could notice anything. When that led nowhere, I stared at the columns until I saw that Locker #1, #4, #9, #16, #25 were each altered an odd number of times, which explained why these lockers remained open at the end. For example, Locker #9 was altered three times (Open to Closed to Open), and similarly Locker #16 was altered five times. But it wasn't clear to me why it worked this way, and why the other lockers, the non-perfect squares, had to have been altered an even number of times.

Mr. Collins jumped in. "Let's look at one of the lockers, say #16. How many times was #16 touched?"

"Five times."

"Which five people altered this locker?"

"Student #1, #2, #4, #8, #16," I said, pointing to my grid.

"Now let's look at Locker #12. Which students altered this locker?"

"Student #1, #2, #3, #4, #6, #12."

"Do you see the pattern?"

I nodded. Instead of just focussing on how many times each locker was altered, I needed to figure out which students altered each locker.

"Okay, now tell me which students touched Locker #20. But this time, don't look at your table."

"Student #1, #2, #4, #5, #10, #20," I said, listing all the *divisors* of the number twenty. Each student, whose locker number evenly divided into twenty, had to alter Locker #20 when they went down the hallway.

"And how about Locker #17?"

"Since seventeen is a prime number, it's just Student #1 and #17."

"Excellent. You've figured out the pattern. Since the number twenty has six divisors, this locker remains closed at the end, since it's altered by an even number of students. But the number sixteen has five divisors, so this locker remains open at the end, since it's altered by an odd number of students. Bethany, can you explain why perfect squares must have an odd number of divisors and non-perfect squares must have an even number of divisors?"

I hesitated. The pattern made sense, but I couldn't explain it.

"Remember how you solved the staircase problem in Mrs. Ridley's Grade 5 class?"

I looked up in surprise. "You know about that?"

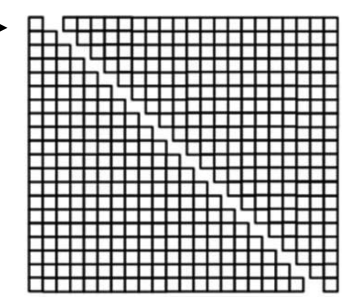
"As you know, Cape Breton is a small place. Sydney is even smaller. And the math teacher community in Sydney is even smaller than that. I ran into Mrs. Ridley shortly after you solved that problem in her class, and she was glowing as she told me about Bethany's Staircase."

"Cool," I said, smiling.

"The staircase," continued Mr. Collins, re-focussing my attention. "Mrs. Ridley asked you to sum up the integers from one to twenty. How did you do that?"

I drew a sketch of the two staircases and explained how to stick them together to form a rectangle. I explained how each row contained twenty-one squares.

"Excellent. I love that solution. Let's unpack that further. Your solution works because you paired each number." He pointed to the first row, with the one square on the top row of my first staircase next to the twenty squares from the bottom row of my inverted staircase.



"In the first row, you've got one square in the first staircase and twenty squares in the second, which adds up to twenty-one. In the second row, you've got two squares and nineteen squares, which also adds up to twenty-one. Every number is paired: one is paired with twenty, two is paired with nineteen, and so on."

I nodded, seeing how each pair added up to twenty-one: (1, 20), (2, 19), (3, 18), and so on.

"Let's get back to the locker room problem," said Mr. Collins. "I want you to pair each number. For example, let's look at Locker #20. You said that this locker was altered by six students. What should the three pairs be?"

I wrote the six divisors of twenty on a piece of paper, namely 1, 2, 4, 5, 10, and 20. Like the staircase problem, I paired the numbers from the endpoints, working in. So my pairs were (1, 20), (2, 10), and (4,5).

"Excellent. What do you notice about each pair?"

"They multiply to twenty."

"Right. Locker #20 remains closed at the end because twenty has an even number of divisors. We have an even number of divisors since we can group the divisors into pairs. To express it in another context, think of a wedding party where all the guests are married and each person comes with their spouse. Since each person comes with their partner, there must be an even number of guests at the wedding. Does that make sense?"

I nodded. In the wedding party context, one would be "married" to twenty, and similarly two and ten would be a couple, as would four and five. That explained why an even number of people touched Locker #20.

"Now do the same with Locker #24. What are its pairs?"

I wrote down the eight divisors of twenty-four, seeing that the four pairs were (1, 24), (2, 12), (3, 8), and (4, 6).

"Great. One more. What about Locker #16?"

I wrote down the five divisors of sixteen, namely 1, 2, 4, 8, and 16. I got two pairs (1, 16) and (2, 8). But then one number was left over. The only way I could pair four was if I could pair it to itself, to produce (4, 4). But that was illegal since there was only one Student #4.

I saw that $16 = 4\times4$. So that explained why perfect squares behaved differently.

"Good, Bethany. From your reaction, it's clear you've got it. So let's answer the original question posed at the very beginning. You have 450 students and 450 lockers. How many lockers remain open at the end, and which ones are they?"

"All the lockers whose numbers are perfect squares: 1×1, 2×2, 3×3, and so on. Since 21×21 is 441, and that's just under 450, there are twenty-one lockers that remain open at the end. The lockers that remain open are 1, 4, 9, and so on, all the way up to 441."

"Well done, Bethany. Let me recap what you just did. You learned another powerful problem-solving strategy today, of *simplifying a difficult problem in order to find a pattern*, which you could then apply to solve the original problem. By pretending there were only twenty-five students at Pinecrest instead of 450, you discovered the pattern of the perfect squares. Once you recognized the pattern, you could then figure out how to explain and solve the harder problem.

"Simplification is a great strategy for real-world problem-solving, whether it's building a small model of an airplane before creating the real thing, or making just a couple of pancakes to figure out the correct proportion of flour to baking soda before cooking a huge batch to feed an army.

"Now, Bethany, here's my challenge for you. Sometime this week, write up a full solution to the locker problem with 450 students, clearly describing the process you just followed to arrive at your final answer. We'll go over your proof next week."

"So you want me to write up what I just explained?" I asked.

"Yes," replied Mr. Collins. "You did a great job explaining it orally; what I want now is a written justification. Your math class needs to emphasize proof-writing a lot more than it does now. I argued for years with the textbook writers and math consultants but got nowhere with them."

"But aren't you now in charge?"

"Yes," he said. "Now that I'm heading up the math curriculum, I'll have the chance to re-shift the focus for our students, with more emphasis on mathematical communication. Say, in all your years in math classes, have you ever written up a proof to a problem you solved?"

"Just once," I said. "The staircase."

"Right. But that was something you did on your own, not a deliberate skill that's practiced and developed as part of a school curriculum. This is why students go off to college or university and can get most of the answers, but they can't explain how they arrived at those answers, nor can they rigorously justify why their calculations are correct. Society doesn't benefit if we graduate engineers and programmers and laboratory scientists who can get the right answers, but can't communicate it to anyone who might be analyzing their reports or de-bugging their computer code or reading their research publications."

"So you're training me to become an engineer or programmer or scientist?"

"No, Bethany. I'm helping you develop your problem-solving skills, as well as your oral and written communication skills. This will all come in handy for whatever career you end up choosing for yourself."

"And what is that?"

He smiled. "Anything you want to be."

"Anything? I can have *any* career I want, as long as I put my mind to it and work hard to achieve it?"

"I believe that with all my heart."

"You're saying that I can be a famous soprano singer? Or a world-champion Olympic gymnast?"

Mr. Collins paused. He looked at me for a few seconds and broke out into a wide grin.

"Okay. Almost any career you want."

7

"S-H-E-P-H-E-R-D."

"That is correct," said Miss Carvery, our vice-principal. The students and teachers in the auditorium applauded as Gillian strolled back to her seat on stage. She passed by Vanessa and gave her a high-five. As Gillian walked by me, she calmly tossed her hair back, lifted her chin up, and gave me a cold stare.

I looked away.

It was now Vanessa's turn. After hearing her word, Vanessa hesitated before slowly responding.

"C-E-M-E-T-A-R-Y."

A loud bell went off.

"I'm sorry, that is incorrect. The correct spelling is C-E-M-E-T-E-R-Y."

I knew how to spell that word, but for all the wrong reasons.

Grandpa passed away two weeks ago.

I sighed, thinking about Grandpa. I missed him a lot. And of course, Mom did too.

After Vanessa walked off the stage, a boy named Rodney made a mistake on the word "deductible", and he too was eliminated. We were now down to the final two spellers. I needed to re-focus.

I took a deep breath, stood up from my seat, and stepped up to the microphone at the centre of the stage. I could see hundreds of people staring at me.

After Miss Carvery gave me my word, I let out a sigh of relief.

"N-E-C-E-S-S-A-R-Y."

The audience applauded. I sat back down, next to Gillian, with neither of us acknowledging the other.

"We have now completed the seventh round of the inaugural Pinecrest Junior High spelling bee," said Miss Carvery. "Congratulations to all of you who participated today. After starting with fifteen spellers, we are now down to our two finalists, Gillian Lowell and Bethany MacDonald. One of these two Grade 7 students will represent Pinecrest at next month's provincial spelling bee in Halifax. The winner of the provincials will represent Nova Scotia at the CanSpell National Spelling Bee in Ottawa."

She looked at us. "Round eight will begin. Gillian, it's your turn."

I glanced over at Gillian, who nodded confidently and walked up to the microphone. I noticed that my hands were wet, and I could feel the sweat on the back of my neck. My legs were shaking.

Though I read many books and was a strong speller, this Spelling Bee was 100% stress and 0% fun. But as I stared at Gillian's back, I knew that I didn't want to lose – to her.

My thoughts were interrupted as Gillian began spelling her word.

"K-E-R-O-S-E-N-E."

It was my turn. Feeling a bit queasy, I relaxed when I heard my word.

"P-A-R-A-L-L-E-L."

Gillian was next. "R-H-Y-T-H-M."

I paused slightly on my next word, but got it. "K-H-A-K-I."

Gillian didn't miss a beat. "U-K-U-L-E-L-E."

Miss Carvery said a word I had heard before, but wasn't sure how to spell. I asked her to say the word again. She leaned into the microphone and slowly enunciated the four syllables: *ko-rah-lay-t*.

Seeing a crowd of people staring at me, I closed my eyes and paused.

Was it one *R* or two *R*'s? Was it spelled with an *A* or an *E*? I visualized the four possible options for the correct spelling, putting each one on its own index card:

CORALATE, CORRALATE, CORELATE, CORRELATE

"Can you say the word again?" I asked.

Miss Carvery repeated the word twice. I still didn't know whether the ending was RALATE or RELATE.

"Can you use it in a sentence?"

She did, but that didn't help. I had one more lifeline.

"Can I have the definition?"

"To place in, or bring into, mutual or reciprocal relation."

As soon as I heard the word *relation*, I knew the word had to end in "RELATE". I was now certain the correct spelling had to be CORELATE or CORRELATE.

But which one? Was it one *R* or two *R*'s?

CO + RELATE looked too easy for it to be the right spelling. COR + RELATE seemed more likely, especially as I knew many words with two R's, like CORRUPT, CORRECT, and CORRODE.

I leaned into the microphone. "C-O-R-R-E-L-A-T-E."

"That is correct."

I smiled and walked back to my seat.

It was now Round 11. Miss Carvery gave Gillian a word I'd never heard. It sounded like *sigh-key*.

Gillian hesitated and took her time before carefully saying each letter.

"P-S-Y-C-H-E."

"That is correct."

"Yes!" said Gillian, as she pumped her fist and returned to her seat, flipping her hair again.

I walked up to the front of the stage. When I heard my word, I calmly nodded. Just to be sure, I asked Miss Carvery for the definition.

This was an easy word I first learned in Grade 1 when we were studying colours. Just to be sure, I visualized the two possible ways the word could be spelled:

MAROON   MARROON

Was it one *R* or two *R*'s? I pictured similar words that had two pairs of consecutive double letters, like RACCOON and BUFFOON and BASSOON, and of course, BALLOON. Yes, it definitely had two pairs of double letters.

"M-A-R-R-O-O-N."

The loud bell went off. I looked at Miss Carvery in disbelief.

"I'm sorry, that is incorrect. The correct spelling is M-A-R-O-O-N."

Shaking my head, I walked over to the other side of the stage, to join my friends Bonnie and Breanna and the other students who had also been eliminated. I heard the audience clapping for me and felt someone pat me on the back.

How did I miss that?

Feeling dazed, I could barely hear Miss Carvery invite Gillian up to the front. I stared blankly at my shoes, annoyed at myself for making such a careless mistake.

"Gillian, this is the championship word. Are you ready?"

"Yes," she replied.

Miss Carvery said the final word. Based on Gillian's excited reaction, I knew she would get it.

"C-A-T-A-C-L-Y-S-M."

"That is correct. Congratulations, Gillian Lowell, you are the winner of Pinecrest's first-ever Spelling Bee!"

The audience stood up and cheered, as Gillian raised both arms in the air. The fourteen other contestants on stage got up from our seats and began to clap.

As Gillian received a shiny trophy and posed for pictures with Miss Carvery, a few students turned towards me.

"Good try, Bethany."

"I was pulling for you."

"Sorry about the last word."

I thanked my classmates, but it was of no comfort.

I turned to see Gillian celebrating with Vanessa and their friends, and exchanging high-fives. A few teachers walked up to Gillian to shake her hand.

After thirty seconds of watching this, I was ready to leave. I turned back to my friends.

"Ready to go?"

"Yeah," said Bonnie. "Let's roll."

Before the Spelling Bee began, the three of us had decided that no matter who won, we were going to treat ourselves to a chocolate sundae at McDonald's. We had lots of time to walk across the street to get our dessert before the school bus came to take us home.

"Wait," I replied. "Just give me one second."

Leaving my friends behind, I walked towards Gillian. As she held up the trophy and celebrated with her clique, I decided I should do the honourable thing. When Gillian saw me, her smile turned into a frown.

"What do you want?"

I held out my hand. "Congratulations. You deserved it."

She looked at my hand and ignored it.

"Maroon?" she snapped. "We learned that in Grade 1. Are you really that dumb?"

Vanessa cackled out loud. She looked at me and rolled her eyes. The Chinese twins had a sympathetic look on their faces, but I knew their allegiances were with Gorgeous Gillian and not with Big Ugly Bethany.

With a clenched jaw, I walked back towards Bonnie and Breanna. "Yeah, I'm ready."

The three of us walked towards the auditorium exit. Just as we got to the door, I felt someone tap me on the shoulder. I turned back and saw Miss Carvery looking up at me.

"Bethany, can I talk with you?"

"Sure," I replied, turning to face the vice-principal.

Miss Carvery looked at Bonnie and Breanna.

"Girls, just head on to wherever you're going. Bethany will meet you there."

"Yes, Miss Carvery."

As Bonnie and Breanna left the auditorium, Miss Carvery led me towards the far end of the auditorium, away from the crowd of teachers and students.

"Please have a seat," she said, pointing towards the seat next to her.

I sat down and looked at my favourite teacher at Pinecrest. She had light black skin and her hair was clipped short, just above the ears. Bonnie's mother told us that Miss Carvery had won a huge scholarship to a famous university in England, and returned back home to Cape Breton to start teaching eight years ago. I couldn't believe that Miss Carvery was only thirty-three years old – a year younger than Mom.

"Bethany, I'm sorry about what happened at the end there."

"Yeah," I sighed. "Such an easy word – I don't know why I spelled it with two *R*'s."

"That's not what I was talking about."

I looked at her in surprise.

"I admire how you handled yourself just now, that even though you were disappointed at finishing second, you had the dignity to walk over and congratulate the winner. I heard what Gillian said to you, and I was disappointed by her actions. But I wanted to commend you for what you did, and the integrity you displayed by calmly walking away. I applaud that."

"Thank you," I replied.

"Other than the last word, did you enjoy the Spelling Bee?"

I swallowed. "Can I be honest with you?"

"Yes, of course."

"I hated it, Miss Carvery. All that pressure, all that tension. It was too competitive. I wish we didn't have to keep going and going until there was only one person left."

"Thank you for your honesty. I agree that the Spelling Bee was competitive, like what happens when hundreds of people are competing for a prize that only one person can win – such as a prestigious entrance scholarship to a university, an executive leadership position in a company, or a spot on the Canadian Olympic team. But I believe today's Spelling Bee served much deeper purposes."

"Such as?"

"To celebrate excellence in academic achievement. To encourage young people to develop their skills in spelling and language through friendly competition. To make people stronger in the face of . . ."

"But it's not friendly competition," I said. "Everyone who didn't win feels like a loser."

"Do you feel like a loser, Bethany? You correctly spelled ten difficult words, without a single error, in an extremely challenging high-pressure environment, with hundreds of people watching you. That's not the mark of a loser. That's the mark of a champion. I know that today's second-place result was hard for you. But you have strong character, and you'll bounce back. I know you will."

"Thank you."

Miss Carvery paused and gently put her hand on my arm.

"I heard that your grandfather passed away. I'm so sorry for your loss. How are you and your mother doing?"

"Okay, I guess. We miss him a lot."

"Of course you miss him. How is your mother doing? I mean, really doing?"

"She's fine," I said, pausing at the question. "Mom hates her job but she isn't unhappy."

Miss Carvery smiled. "I hear she's been coaching figure skating for the past couple of months."

"Yes," I said, surprised she knew. "She coaches an eight-year old on Saturday afternoons."

"The granddaughter of Taylor Collins," said Miss Carvery, nodding. "I know, since I ran into Mr. Collins the other day. He's delighted to be working with you. Both you and Ella are fortunate to have found such excellent coaches."

"Thanks," I said. "I'm very lucky. Ella too."

"Though I admit I was shocked to hear that your mother agreed to come back to the sport as a coach."

"Why?" I asked, confused. "Wasn't she an amazing figure skater?"

"Of course she was," replied Miss Carvery. "Everyone in Cape Breton knew Lucy MacDonald. All of us used to cheer for her."

"But you just said you were shocked that Mom would coach figure skating. Of course she would. Mom was the best figure skater in Nova Scotia three years in a row!"

Miss Carvery looked at me strangely. After a few seconds of uncomfortable silence, she gasped and put a hand to her mouth.

"What's wrong?" I asked.

"Nothing's wrong," said Miss Carvery, quickly moving her hand back on to her lap. She paused and put her hands on my shoulders.

"You should go, Bethany. Your friends are waiting for you."

8

"My best score ever," I said, passing a sheet to Mr. Collins.

Mr. Collins smiled as he checked my answers to the twenty-five questions from a previous Grade 7 math contest. The exam had taken me seventy-five minutes to complete, exceeding the one-hour time limit, but this was for fun – and not for competition.

The Grade 7 paper was organized by the University of Waterloo and was called the "Gauss", named after some famous German mathematician. The Gauss contest was written by nearly twenty thousand students each year from all across Canada.

"Yes, Bethany, I think this might be your best result yet," said Mr. Collins, comparing his multiple-choice responses to mine. "Your first nineteen are all correct."

In January, Mr. Collins and I had begun a new routine, where we spent some time at the beginning of each session reviewing the problems from an old Gauss contest. While I decided that I wouldn't be writing the actual contest with other Pinecrest students in May, I really enjoyed these contest problems – they were a lot more interesting than the stuff we covered in Grade 7 Math. And because it wasn't a high-pressure contest environment, I could take as long as I wanted, and not have to rush through the problems. Mom often told me how I had the right perspective: doing math because it was fun, rather than competing or comparing myself with Gillian Lowell or anybody else at Pinecrest.

"Any ideas on #20?" asked Mr. Collins, noticing the one question I had left blank.

"Not really. I got stuck because there were too many words. There was too much stuff to keep track of."

"Hang on one second, Bethany. I'll be right back."

Mr. Collins stood up, walked over to the counter at the front of Le Bistro, and returned a few seconds later with ten packets of sugar.

"What's this?"

"Hold these packets in your hand, so that you have something concrete to work with. It will be much easier to solve problems like this if you use props."

"Isn't that cheating?"

"Not at all," he said, smiling. "It's creative problem-solving."

Taking the ten packets of sugar in my hand, I looked at the one question I had left blank.

20. Anne, Beth and Chris have ten candies to divide amongst themselves. Anne gets at least three candies, while Beth and Chris each get at least two. If Chris gets at most three, the number of candies that Beth could get is

(A) 2 (B) 2 or 3 (C) 3 or 4 (D) 2, 3, or 5 (E) 2, 3, 4, or 5

Anne got at least three packets of sugar, while Beth and Chris got at least two. So I put three packets on my left, two in the middle, and two on my right, keeping the remaining three in my hand. I wanted to know how many total packets could be placed in the middle, the pile belonging to Beth. Of the three remaining packets, how could I distribute them?

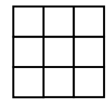
I suddenly realized that the information about Chris getting at most one extra packet was unimportant. Since Anne and Beth could be given additional packets with no restriction, I could just distribute the remaining three packets between Anne and Beth. So Beth could receive zero, one, two, or three additional packets of sugar, and Anne could take the rest:

Scenario	Anne	**Beth**	Chris
Beth gets zero additional packets	6	**2**	2
Beth gets one additional packet	5	**3**	2
Beth gets two additional packets	4	**4**	2
Beth gets three additional packets	3	**5**	2

Yes, that worked for sure. I looked at the five possible answers on the contest sheet, and saw that option (E) corresponded to Beth getting 2, 3, 4, or 5 candies. I circled (E).

"Correct," replied Mr. Collins. "Question #21 is straightforward, and we've covered problems like that many times. So let's move on to Question #22. Can you show me how you did this one?"

22. The total number of squares and rectangles, of all sizes, that appear in a 3×3 square is



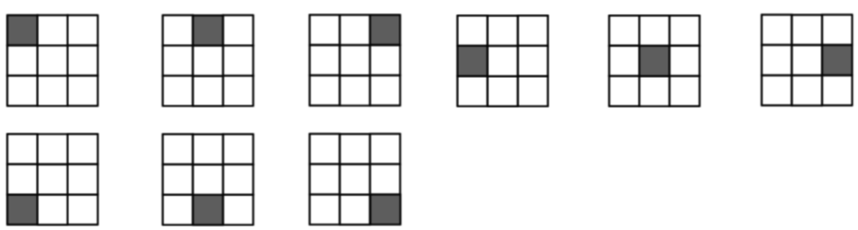
(A) 28 (B) 30 (C) 32 (D) 34 (E) 36

"I broke the problem down into cases, turning a hard question into a bunch of smaller simpler questions. I figured out all the types of squares and rectangles that could appear, you know, 1×1, 1×2, 2×2, 2×3, and so on. I counted each case, and then added it all up."

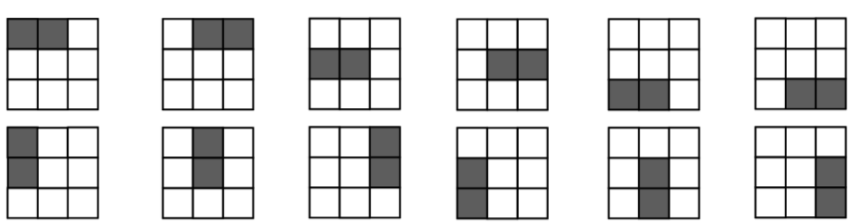
"Excellent," replied Mr. Collins. "You simplified the problem by figuring out the possible dimensions of the squares and rectangles that could appear in the picture, and then found its total. What did you get?"

I handed him a sheet of paper with all the different possibilities.

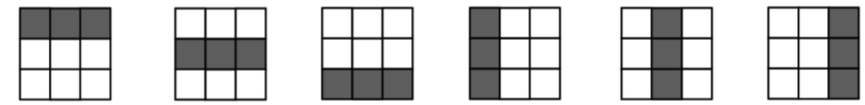
There were nine 1×1 squares, since there were nine unit squares in the diagram.



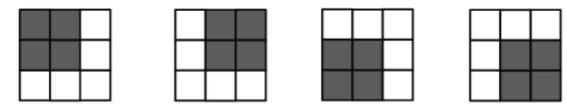
As for 1×2 rectangles, there were twelve of them in the picture: six "horizontal" and six "vertical".



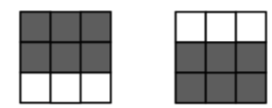
Counting 1×3 rectangles, I found six.



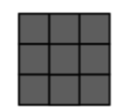
Counting 2×2 squares, I found four.



Counting 2×3 rectangles, I found two.



Finally, there was the 3×3 square. Of course, there was only one.



"So I found all the cases, and added them up. I got 9+12+6+4+2+1, which adds up to 34. So that's the answer."

Mr. Collins paused. "Are you sure you considered every case?"

"Yes," I said, before hesitating. "I think so."

Mr. Collins picked up a piece of paper and proceeded to draw a little table.

Column Dimension

Row Dimension

	1	**2**	**3**
1			
2			
3			

"Let's look at the row and column dimensions of each possible rectangle that can appear in our big 3×3 square. I want you to summarize this information in this table. Note that in any rectangle, its row dimension or column dimension can't be more than three, since the big square is only 3×3. Do you agree?"

I nodded. We couldn't have a 4×5 rectangle inside a 3×3 square, nor could we have rectangles with dimensions such as 2×4 or 5×1. Both the row and column dimension could be at most three.

"Bethany, I'd like you to put an **X** mark next to every scenario you've considered. For example, you counted twelve rectangles that were either 1×2 (horizontal) or 2×1 (vertical). Mark both of those entries in my table with an **X**. Do the same with all the other cases."

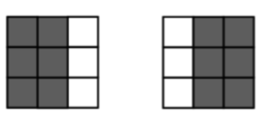
Looking at my diagrams of shaded squares and rectangles, I put an **X** next to every case I had considered.

Column Dimension

Row Dimension

	1	**2**	**3**
1	X	X	X
2	X	X	X
3	X		X

Once I completed the table, I realized I had forgotten to consider 3×2 rectangles. It was easy to see that there were two of them.



So the total number of squares and rectangles wasn't 34, but 34+2.

"The answer is thirty-six."

"That's right," said Mr. Collins. "To answer this difficult problem, you simplified it into smaller cases, which in this context, was to enumerate the squares and rectangles of all possible dimensions."

"Enumerate?" I asked.

"Sorry. Enumerate is just a fancy word for count."

"I see."

"You broke the problem into different cases and enumerated each case separately. You looked at 1×1 squares, then 1×2 rectangles, then 2×1 rectangles, and so on. But what happened?"

"I forgot a case."

"That's right. You were convinced that you had found all the possible cases, but once you summarized this information in your little table, you realized you had forgotten the 3×2 rectangles. You discovered this only because you presented the information in a highly structured way.

"When you're breaking down a hard problem into smaller simpler cases, you need to figure out how to structure your information so you can be certain that you haven't forgotten anything. Having a clearly-defined structure is the difference between *thinking* you've considered every case, and *knowing* you've considered every case. Say, that reminds me. Can I tell you a story?"

"Yes, please."

"Have you ever heard of the Challenger space shuttle disaster, the one from 1986?"

I shook my head. That was way before I was born.

"It was a terrible tragedy. The American space shuttle Challenger exploded seventy-three seconds after takeoff, killing all seven astronauts on board. NASA had never launched a shuttle with the temperature below ten degrees Celsius, but they let Challenger launch on a day when the temperature was only two degrees. They figured the cold weather wouldn't affect any part of the shuttle. But they overlooked something.

"The rocket boosters of a space shuttle are comprised of several parts, which are fitted together by giant rubber bands called O-rings, to ensure air doesn't escape. In the cold weather, these O-rings didn't seal properly, allowing pressurized hot gas to escape the rocket booster and reach the external fuel tank. That created a deadly combustion effect, and the space shuttle exploded."

"Did anyone know about these O-rings?" I asked.

"Yes, people did know, and that's what makes the disaster even worse. The NASA engineers knew the O-rings would work properly as long as the temperature remained above ten degrees Celsius. But once the temperature dropped to freezing, they predicted the O-ring would lose resilience, that it would be unsafe to launch. The engineers reported the problem to their higher-ups, but they were overruled."

"Overruled?"

"The mission commanders at NASA, the big guns making the decisions, insisted the engineers prove it was *not* safe to launch rather than demonstrate the conditions *were* safe to launch. Notice the difference between the two. The engineers couldn't show that it was not safe to launch as they had never tested the O-rings in such cold temperatures.

"During one of the public hearings in the aftermath of the disaster, a Nobel-prize winning physicist took a small O-ring, and placed it in some ice water for a few minutes. When he took out the O-ring from the ice water, he showed how it lost elasticity and became brittle."

"But I don't get it," I said. "Why did they launch if they knew something could go wrong?"

"Well, NASA was under a lot of stress. The space shuttle launch was delayed a bunch of times that previous week, because of bad weather. Everyone was waiting for the launch to occur, and people were getting impatient due to the weather delays. So despite the objections and concerns from the engineers and other scientists, the mission control at NASA decided to lift off, with horrible consequences. It was a sad case of public relations trumping scientific and engineering reality.

"The moral of the story is the importance of performing rigorous scientific analysis. In the real world, a single omission can lead to disaster. Even if you remember ninety-nine out of one hundred items, it's not enough, especially when the one item you forget is the O-ring. Through the process of writing mathematical proofs, you're learning how to present arguments that are air-tight, where every line follows logically from the previous line, with no holes in your reasoning. Bethany, you'll learn never to forget the O-ring as you master a subject that's never bo-ring."

I laughed out loud.

"Let's continue. Of the first twenty-two questions on the practice Gauss contest, you solved twenty of them on your own, and together we figured out the other two. Take me through the last three."

We went through the final three problems, and I was delighted when Mr. Collins found no holes in any of my arguments.

"Wonderful, Bethany. So how many questions did you correctly solve?"

"Twenty-three out of twenty-five."

"What's that as a percentage?"

I multiplied twenty-three by four in my head. "That's ninety-two percent."

"That's right. That's your best score so far. I went online this morning to check the statistics on that contest. Of the twenty thousand Canadian students who wrote the Gauss that year, did you know that fewer than one percent solved twenty-three or more questions? Bethany, you have become an outstanding mathlete!"

I smiled, realizing that one percent of twenty thousand was just two hundred.

"But I can't compare myself to them," I said, remembering that those two hundred students had to write the contest under pressure. "I had all the time I wanted this morning. They only had an hour."

"Good point. But if you were presented with the same circumstances, I bet you would do quite well."

"I don't think so."

"Why do you say that, Bethany?"

"Because I know myself. I don't do well under pressure. Remember the Spelling Bee?"

"Don't be so hard on yourself. As you told me, you were eliminated in the eleventh round. That means you spelled ten out of eleven words correctly."

"But I lost."

"No, you didn't," said Mr. Collins. "You misspelled a single word, and came in second place."

"Second place is the first loser. Someone at Pinecrest had that on their T-shirt."

"Oh, Bethany, don't think of it that way. That T-shirt slogan sends a horrible message, especially to young people. And while you are naturally disappointed in finishing second, I want to encourage you to think of it a different way: it was just you and Miss Carvery that day. She gave you eleven words to spell and you correctly got ten of them. That's over ninety percent. And maybe next year, when you're in Grade 8, you'll get an even higher percentage right."

"I won't be doing the Spelling Bee next year."

"That's fine, and I respect your decision. If the Spelling Bee isn't fun for you, then there's no sense entering it next year. It's the same with this Gauss Contest. If you find the problems interesting and want to do the contest because you find math enjoyable and challenging, then I'd encourage you to participate with the other Pinecrest students in May. But if it only leads to stress and anxiety, and you find yourself competing against Gillian Lowell and feeling bad if she solves more questions than you, then you're right – it's better that you don't participate."

I nodded.

"Bethany, let me change the subject for a minute. I'm sixty-three years old. I was never a star athlete but I always enjoyed running. And about twenty years ago, I started running half-marathons, gradually building up to full marathons."

"You've run a marathon?" I said in surprise.

"Yes. In fact, I've run five of them. There's a big race in Halifax each May, called the Bluenose Marathon, and this May will be my sixth time running that course. My goal each year is to simply beat my personal best time. I'm not interested in how many people I beat, or what place I finish. It's just me versus the clock. Last year I ran it in four hours and two minutes, forty-two kilometres on a cold rainy day."

"That's amazing you can run for over four hours."

"Thank you," replied Mr. Collins. "It's taken years of training, pushing my body to see how far and how fast I can go. My target this May is to run the marathon in just under four hours, which would be a first. And I think I have a good chance, especially if I'm not running in a rainstorm."

"Do you enjoy it?" I asked, wondering how anyone could possibly enjoy long-distance running.

"I love running, especially when I'm with my Sunday morning group along the Sydney Waterfront. Of course, they're all younger and faster, but that doesn't bother me. I feel so alive when I run. I can tell how alive you feel when you do math. What I get from running is what you get from math."

"Can I ask you a question?"

He smiled. "Of course, Bethany."

"I got twenty-three out of twenty-five on this practice Gauss contest, and that's my best score so far. So maybe I should write the actual Gauss contest in May?"

"It depends on your attitude. Tell me why you might want to write the contest."

"To try and beat my personal best of twenty-three questions. To see how much I've improved since September. Just me versus the questions, that's it."

"And what if one of your classmates gets a better score than you?"

I paused.

"Then that's fine. It just means that one of my classmates got a better score. No big deal."

"Are you being honest? If Gillian Lowell solves more questions than you, would you really not care?"

"No," I admitted. "But maybe one day, that wouldn't bother me."

Mr. Collins looked at me and smiled.

"Maybe one day, you and Gillian will be teammates."

9

"Twenty-four out of twenty-five. Well done."

"That's ninety-six percent," I replied, satisfied with my score because I had no chance on the final question.

"That ties your personal best," said Mr. Collins, handing me a sheet of paper filled with a bunch of numbers. "With the Gauss contest less than a week away, this is a great sign. You're definitely ready."

"Thanks, Mr. Collins. Can you tell me what these numbers are?"

"You tell me."

I looked at the sheet of paper more carefully.

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

"They're all the two-digit numbers," I said.

"That's right. I've given you the set of two-digit positive integers. And how many are there?"

"Ninety," I replied instantly, seeing that the numbers on the card appeared in a 9×10 table.

"Now suppose you randomly select a number from this table. What's the probability the last digit is one?"

Looking at the second column of the card, I saw there were nine numbers with last digit one, namely the integers 11, 21, 31, 41, 51, 61, 71, 81, and 91. Since there were ninety numbers in all, the answer was 9/90, which reduced to 1/10.

"One-tenth," I responded.

"Good. Now what's the probability the *first* digit is one?"

This was also straightforward. Looking at the first row of the card, I saw there were ten numbers with first digit one, namely the integers 10, 11, 12, 13, 14, 15, 16, 17, 18, and 19. The answer was 10/90.

"One-ninth."

"Excellent. Now suppose I gave you a card that listed all the three-digit integers. And say I got you to pick a number from that card at random. What would be the probability the first digit is one?"

As the two-digit scenario had probability 10/90, I reasoned that the three-digit scenario would have probability 100/900.

"It's the same," I said. "One-ninth."

"Good. Now how about five-digit integers, or fifty-digit integers, or hundred-digit integers?"

"Exact same – it's always one-ninth."

"What can you conclude? Be specific."

"That if you pick any integer at random, no matter how large it is, there's a probability of one-ninth that the first digit of that number is one."

Mr. Collins gave me another sheet of paper. "Now tell me what this is."

1) China	1,330,000,000	6) Pakistan	177,000,000
2) India	1,210,000,000	7) Nigeria	158,000,000
3) USA	312,000,000	8) Bangladesh	151,000,000
4) Indonesia	237,000,000	9) Russia	142,000,000
5) Brazil	190,000,000	10) Japan	127,000,000

I quickly figured out what these numbers represented.

"They're population numbers."

"Correct. China has approximately 1.3 billion people, followed by India with 1.2 billion people, and so on. This is the list of the ten most populated countries. If you were to pick a country at random from this table, what's the probability that its population has first digit one?"

Eight of the ten countries had their population beginning with the digit one, with the exception of the United States and Indonesia. So the probability was 8/10.

"It's eight out of ten – eighty percent."

"But didn't you say that the expected probability was one-ninth? Eighty percent is a lot more than eleven percent."

"Yeah, but these are special numbers," I said.

"What do you mean by *special*?" asked Mr. Collins.

"Well, you just gave me the populations for the top ten countries, so it was just luck that eight of them had first digit one. If you looked at every country's population, the first digits would be evenly spread out."

"Are you saying that if we looked at population data for all the world's countries, it would be equally likely for the first digit to be one as for the first digit to be nine? That the probability would be about one-ninth for each of the nine possible first digits?"

"Yes."

Mr. Collins smiled and took out some papers from his clipboard.

"Well, let's see, shall we?"

I stared at the three-page Wikipedia printout, listing the approximate population numbers for the top two hundred countries, ranging from 1.3 billion for China (#1) to 56,000 for Greenland (#200).

"How did you know I was going to say that?"

Mr. Collins smiled. "Just a lucky guess."

"What do you want me to do?"

"There are about two hundred and twenty-five countries in the world, if you include small dependent territories like the Falkland Islands and the Cayman Islands. I've taken the top two hundred, just to simplify the calculations. What I want you to do is make a table that lists the number of countries, and the percentage of countries, that have populations beginning with each of the digits from one to nine."

I flipped through the handout, tallying up the first digits for each country's population, and then calculating the proportion of countries with that first digit. Since there were two hundred countries in my list, the percentages were easy to calculate.

First Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Countries	58	28	26	18	17	17	10	17	9
Percentage	29%	14%	13%	9%	8.5%	8.5%	5%	8.5%	4.5%

"Great," said Mr. Collins. "What do you notice?"

"Twenty-nine percent of countries have their population beginning with the digit one, while only four and a half percent of countries have their population beginning with the digit nine."

"So these statistics are not evenly spread out. It's not one-ninth for each of the nine possible first digits."

"But why?"

"Yes, that's exactly what I said when I first learned about this phenomenon. Whenever you have a list of *naturally-occurring numbers*, like the lengths of the world's longest rivers, the heights of the world's tallest buildings, the numbers that appear in your local newspaper, or the house numbers of residents in Cape Breton, it is far more likely for the numbers to have a low first digit than a high first digit."

"I don't believe you."

Mr. Collins handed me another card. "At first, I didn't believe it myself. Here's the magic table that states the distribution of first digits in such naturally-occurring numbers."

First Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Percentage	30.1%	17.6%	12.5%	9.7%	7.9%	6.7%	5.8%	5.1%	4.6%

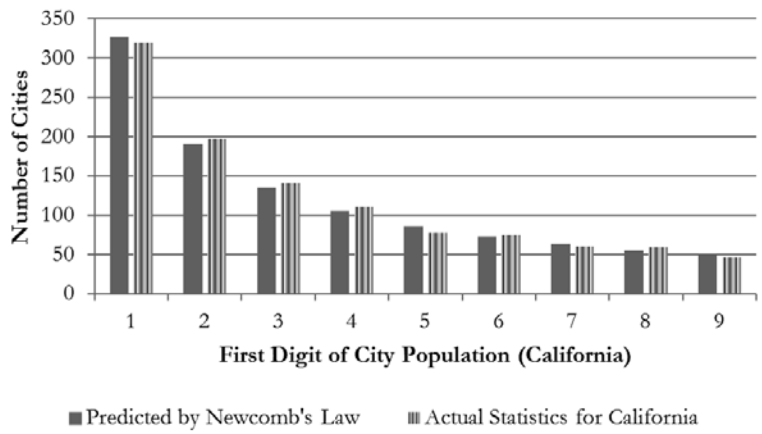
"Notice how these numbers are really close to the ones you came up with for country population?"

I nodded, seeing the similarity between his magic table and my calculations for the population numbers: 29%, 14%, 13%, 9%, all the way down to 4.5%.

Mr. Collins continued. "Let me illustrate with another example that I found on the internet yesterday. California has a whole bunch of cities: huge cities like Los Angeles with 3.9 million people and San Diego with 1.3 million; medium-sized cities like Pasadena with 150,000 people; and tiny cities that you've never heard of with just a few thousand people."

He handed me another sheet of paper.

"If you determine the first digit of each city's population, and tally the results, here's what the graph looks like."



"The actual statistics match up almost identically with what's predicted by Newcomb's Law."

"What's Newcomb's Law?" I asked.

"Sorry," replied Mr. Collins. "That's the name for the magic table I just gave you: 30.1% for the first digit one, 17.6% for the first digit two, and so on."

"Who was Newcomb?"

"I'll get to that in a moment. You know, this first-digit phenomenon isn't just for population, it's also for things like house numbers. If you randomly picked one hundred streets in Cape Breton, and randomly selected one house on each of those streets, then around thirty houses will have first digit one."

"Why?"

"Let me give you a small hint. Think of various streets in Sydney, big ones like George Street and Kings Road, as well as smaller streets like the one I live on. What's the range of the house numbers? Do they all go from 1 to 99, or 1 to 999, or do some streets cut off earlier? For example, I think George Street goes from 1 to 2400, or something like that. Say you live on George Street. Is it more likely that your house number starts with one or nine?"

"I live in an apartment."

"I know," said Mr. Collins, smiling. "My cousin fixes elevators for a living. He says he comes by your building at least once a week."

I laughed, suddenly realizing that the elevator repairman did look a lot like Mr. Collins: bushy grey hair and super-thin, though he had no mustache.

"So, Bethany, let me ask the question again. Say you live in a random building on George Street, a street whose numbers go from 1 to 2400. What's the most likely first digit?"

If I lived on George Street, and the first digit of my house number was nine, my options would be limited to 9, 90 to 99, and 900 to 999. On the other hand, over a thousand houses on George Street had first digit one, specifically everything from 1000 to 1999, in addition to 1, 10 to 19, and 100 to 199.

Breanna lived on a street where the numbers went from 1 to 56. So that meant that there were eleven houses that had one as its first digit (specifically 1, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19), but only one with nine as its first digit.

"I think I understand Newcomb's Law."

"Good," said Mr. Collins. "Explain your ideas to me."

After Mr. Collins agreed with my justification, a question still nagged.

"But how did they come up with the numbers in your magic table?" I asked. "You know, 30.1% of numbers have a first digit of one, 17.6% have a first digit of two, and so on?"

"Take out your calculator and do the following: for each of the numbers you see in Newcomb's table, convert it to a decimal. For example, with the percentage 30.1%, change that to 0.301. Then calculate the value of $10^{0.301}$. Do the same with all the other numbers in the table."

I followed his instructions and produced the following:

First Digit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Proportion	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
$10^{Proportion}$	2.000	1.500	1.333	1.250	1.200	1.167	1.143	1.125	1.111

"Notice anything interesting?"

I saw it right away.

"Those numbers in the bottom row are simple fractions. It reminds me of the telescoping series that you taught me last week."

$$\frac{\mathbf{2}}{\mathbf{1}} = 2 \quad \frac{\mathbf{3}}{\mathbf{2}} = 1.5 \quad \frac{\mathbf{4}}{\mathbf{3}} = 1.333 \quad \frac{\mathbf{5}}{\mathbf{4}} = 1.25 \quad \frac{\mathbf{6}}{\mathbf{5}} = 1.2$$

$$\frac{\mathbf{7}}{\mathbf{6}} = 1.167 \quad \frac{\mathbf{8}}{\mathbf{7}} = 1.143 \quad \frac{\mathbf{9}}{\mathbf{8}} = 1.125 \quad \frac{\mathbf{10}}{\mathbf{9}} = 1.111$$

"That's correct. Way back when, before we had calculators, people had to use something called 'logarithm tables' to do calculations like 123456789 × 654321. It was a thick book with thousands of pages. Using logarithm tables was a tedious process but the calculations had to be precise – and was particularly needed by astronomers for surveying and celestial navigation.

"One day, a mathematician named Simon Newcomb looked at these logarithm tables and noticed that the earlier pages were more worn out than the later pages – in other words, there seemed to be more naturally-occurring numbers with first digits one and two than first digits eight and nine. He wondered why that was. Then he used techniques from a branch of math called 'probability theory' to show that the first digit frequencies had this beautiful property relating to powers of ten and simple fractions."

"That's really cool."

"Yes, it is. And I haven't told you the best part of the story. Guess where Simon Newcomb was from?"

"I don't know," I replied with a shrug.

"Guess."

"America? Britain?"

"He's from Nova Scotia! Simon Newcomb was born in Wallace, right next to Pugwash. It's just a four-hour drive from Wallace to Sydney."

I looked at him incredulously. "He's from Nova Scotia?"

"That's right! Our fellow Nova Scotian, Simon Newcomb, discovered the first-digit phenomenon back in 1881. Fifty years later, an American mathematician named Benford discovered the same principle, and he published it too. For some reason, the first-digit discovery is now known as Benford's Law even though Newcomb discovered it first."

Mr. Collins glanced at his watch.

"Before we end today, I want to summarize what we discussed because it's one of the most fundamental uses and applications of mathematics.

"Simon Newcomb looked at these logarithm tables and noticed the pattern that the earlier pages were more worn out than the later pages. Having found the pattern, he sought to understand and uncover its hidden structure. To do this, he had to apply his problem-solving skills to find a logical explanation for the first-digit phenomenon that 30% of the numbers start with one, about 18% start with two and so on. The mathematics was deep, and the end result was completely unexpected. That's the process by which mathematics is done in the real world. It's not people sitting around memorizing formulas, but rather employing a multitude of techniques to discover patterns to uncover truth and structure, seeing how those patterns explain what we see in society, and propose fresh ideas based on rigorous evidence to change our world.

"Newcomb couldn't have predicted this, but his discovery has led to numerous important and practical applications over a century later. One such application is fraud detection. As you know, your mother works at the Canada Revenue Agency, the federal department that administers all the country's tax laws and processes income tax reports. Well, if you look at a company's tax returns, you'll see thousands of different numbers that represent bits of financial information: assets, profits, stock prices, interest, and so on. If you look at the first digits of these numbers, what do you think happens?"

I saw where this was leading. "The first digits follow Newcomb's Law: 30% should have first digit one, about 18% should have first digit two, and so on."

"Exactly. So a legitimate tax return will have far more entries like $1,805 and $105.36 than those beginning with larger digits, such as $98.35 and $800. On the other hand, those who fudge the data and try to make the numbers appear random tend to distribute the numbers equally, with the first digits spread out evenly between 1 and 9. So using Newcomb's Law, you can identify the criminals and send them to jail."

"But if you knew Newcomb's Law, couldn't you properly fudge the numbers and get away with it?"

"No comment," said Mr. Collins.

"How do you know all this?" I asked, fascinated by the story.

"Because a former student of mine has been working for the Canada Revenue Agency in Ottawa, managing a research group specializing in fraud detection. He does cutting-edge work for the federal government, using math and statistics for risk assessment, saving the Government of Canada millions of dollars each year. It's a wonderful application of mathematics, and so practical."

"Thanks, Mr. Collins. I learned a lot today."

"You're welcome. And good luck on next week's Gauss Contest. Are you excited?"

"Not excited. Nervous."

"You'll do great. Remember you got 96% on today's practice contest. And here's a small tip: if you think you're going to be distracted by a certain classmate and worried about what she's doing, then just sit in the front row. This way, you won't be distracted."

"Okay," I said. "And good luck with the marathon. Hope you break four hours."

"Thank you. By the way, do you know about Gauss? You know, who he was, what he was famous for?"

"Let me guess," I said. "He's from Nova Scotia too?"

"He's from Germany, but good try," said Mr. Collins. "Carl Gauss was one of the world's greatest mathematicians. He made enormous contributions to statistics, astronomy, geophysics, and created new fields of math which were centuries ahead of his time. There's a famous story about him.

"When young Carl Gauss was in Grade 4, his teacher gave all of the students a mindless and tedious addition problem. Of course, this was in the 1700s, and they had no access to calculators. So the students performed the addition term by term, and it took them all a long time. And every single student got the answer wrong, that is, every student except for Gauss. It turns out that Gauss correctly solved the problem, and he did so in mere seconds because he saw an insight that no one else did."

"What was the addition problem?"

Mr. Collins smiled. "It was to find the sum of the integers from 1 to 100. Gauss' incredible breakthrough was to create a second sum, and write the numbers backwards from 100 down to 1."

$$\begin{aligned} S &= 1 + 2 + 3 + 4 + \ldots + 97 + 98 + 99 + 100 \\ S &= 100 + 99 + 98 + 97 + \ldots + 4 + 3 + 2 + 1 \end{aligned}$$

Mr. Collins added the two lines, column by column, which made each term on the right equal to 101.

$$2S = 101 + 101 + 101 + 101 + \ldots + 101 + 101 + 101 + 101$$

"The left side is 2S, twice the desired sum. The right side is 101×100, since there are 100 terms in total. Therefore, $2S = 101\times100$, which implies that $S = 101\times50 = 5050$. Gauss blurted out the answer in seconds. His classmates were amazed, and his teacher was astounded by his creativity and imagination. It was the first sign of his talent in mathematics, an ability that he nurtured and developed to become arguably the most important mathematician in the history of the world."

My mouth went dry.

"Don't worry, Bethany. It's not like I'm not comparing you to Gauss."

Mr. Collins winked.

"After all, you only had to add up the numbers from 1 to 20."

10

"The contest will begin in three minutes."

Our vice-principal Miss Carvery walked around the room, handing out a green sheet of paper to all of us.

I was the only person sitting in the first row. There were a dozen students writing the contest, including Gillian and Vanessa, who were sitting next to each other two rows behind me.

I arranged everything I needed so it was right in front of me: three pencils, fifteen sheets of white paper, an eraser, a ruler, a compass, and a calculator.

The green answer sheet contained twenty-five rows, one row for each multiple-choice question, with five possible answers: (A), (B), (C), (D), (E). Miss Carvery explained how each answer was to be bubbled in using pencil, and that we had exactly sixty minutes to answer the twenty-five problems: ten easy questions in Part A, ten medium questions in Part B, and five hard questions in Part C.

"Okay, it's now eleven o'clock. You can start. Good luck!"

I opened the contest booklet and took a deep breath.

Here we go.

1. When the numbers 8, 3, 5, 0, 1, are arranged from smallest to largest, the middle number is

(A) 5 (B) 8 (C) 3 (D) 0 (E) 1

The answer was 3. I moved my pencil to the first row in my answer sheet and bubbled in (C).

2. The value of 0.9 + 0.99 is

(A) 0.999 (B) 1.89 (C) 1.08 (D) 1.98 (E) 0.89

My first instinct was to bubble in the answer 0.999 but then I looked at the numbers more carefully and saw that the correct answer was actually 1.89, since ninety cents and ninety-nine cents added up to $1.89. I knew that I had avoided a potential trap, and bubbled in the correct answer, (B).

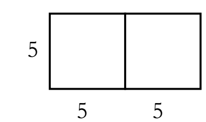
Five minutes later, I was at Question #10.

10. Two squares, each with an area of 25 cm² are placed side by side to form a rectangle. What is the perimeter of this rectangle?

(A) 30 cm (B) 25 cm (C) 50 cm (D) 20 cm (E) 15 cm

I drew the diagram in the contest booklet. A square with area 25 cm² has side length 5 cm. So when the squares are placed side by side to form a rectangle, the width is 5 cm and the length is 10 cm. The perimeter is 5+10, which equals 15 cm.

I bubbled in (E).



I was on a roll.

13. A *palindrome* is a positive integer whose digits are the same when read forwards or backwards. For example, 2002 is a palindrome. What is the smallest number which can be added to 2002 to produce a larger palindrome?

(A) 11 (B) 110 (C) 108 (D) 18 (E) 1001

I recalled Mr. Collins' advice that when it's not clear how to solve the problem directly, start from the multiple-choice answers and work backwards. This was just one of the many strategies I called "cheating" and he called "creative problem-solving". I read the question again, finding the key word.

What is the *smallest* number which can be added to 2002 to produce a larger palindrome?

I started with the smallest number among the five multiple-choice options, and worked my way up. As soon as I found a palindrome, I would have my answer.

2002 + **11** = 2013. Not a palindrome.
2002 + **18** = 2020. Close to a palindrome, but not quite.
2002 + **108** = 2110. No.
2002 + **110** = 2112. Yes!

With a smile on my face, I bubbled in (B). I glanced at the clock, noting that the time, 11:11, was also a palindrome.

14. The first six letters of the alphabet are assigned values *A*=1, *B*=2, *C*=3, *D*=4, *E*=5, and *F*=6. The value of a word equals the sum of the values of its letters. For example, the value of *BEEF* is 2+5+5+6 = 18. Which of the following words has the greatest value?

(A) *BEEF* (B) *FADE* (C) *FEED* (D) *FACE* (E) *DEAF*

I recalled another strategy from Mr. Collins, to take a few seconds right at the beginning of each question and look for an insight that would make the problem easier and less mechanical.

All of a sudden, I saw what to do: each of the five words had the letters *F* and *E* appearing, so these would effectively "cancel" each other when determining which of the five words had the greatest sum. I realized that finding the greatest value of these five words:

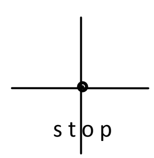
(A) *BE**EF*** (B) ***F**AD**E*** (C) ***FE**ED* (D) ***F**AC**E*** (E) *D**E**A**F***

was equivalent to finding the greatest value of these five "words":

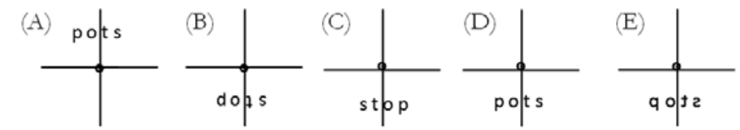
(A) *BE* (B) *AD* (C) *ED* (D) *AC* (E) *DA*

From here, I could tell that *ED* = 5+4 = 9 had the greatest value. The answer was (C). The questions got progressively more difficult.

20. The word "stop" starts in the position shown in the diagram



It is then rotated 180 degrees clockwise about the origin, and this result is then reflected in the *x*-axis. Which of the following represents the final image?



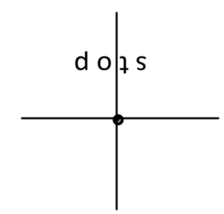
There were four letters to keep track of in my head, in addition to one rotation and a flip. It was too much to hold in my mind.

I sat and pondered for at least one minute.

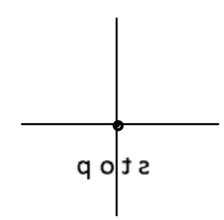
An idea hit me. Remembering Mr. Collins and his sugar packets, I thought of something I could do.

I took out a fresh sheet of paper, drew two large perpendicular lines that filled the entire page, signifying the horizontal *x*-axis and vertical *y*-axis, and wrote down **s t o p** in large block letters with my pencil, making sure the letters were dark enough that I could see through them when I needed to flip the page over.

I held the paper in my hand. First I rotated the sheet 180 degrees clockwise about the origin, which gave me the following picture:



For the reflection, I pinched the left and right side of the paper with my thumbs lying on the same line as my horizontal *x*-axis, and just turned over my wrists. The paper flipped over and I saw the final image by holding my piece of paper to the light above me:



Scanning the five multiple-choice options, I saw the answer I was looking for, and bubbled in (E).

I glanced at the clock, and realized that I had finished the first twenty problems in twenty-four minutes, giving me plenty of time to work on the five hard Part C questions. I quickly checked my answer sheet and was relieved to see that I had bubbled everything in correctly.

Question #21 was a simple problem of counting handshakes; Question #22 dealt with the surface area of a rectangular box; Question #23 was a probability question involving coloured marbles; and Question #24 asked for the area of a trapezoid shaded inside a particular triangle. I was confident that I had correctly solved all four, and was super-careful to check each step.

11:45 a.m.

Fifteen minutes left for the final question.

25. Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?
(A) 4 (B) 18 (C) 24 (D) 12 (E) 21

I knew all the ways we could multiply two integers to get to 24. The combinations were 1×24, 2×12, 3×8, and 4×6. Now how could I adapt this to three integers?

I started listing the options. Taking out a fresh sheet of paper, I wrote down all the ways three digits could multiply to give 24.

2×3×4 2×2×6 1×4×6

That's all I could come up with. I ignored options such as 1×1×24 and 1×2×12, since 12 and 24 weren't digits. All I needed to do was determine how many three-digit integers could be made from each of the sets [2,3,4], [2,2,6], and [1,4,6], and I would be all done!

I knew that [2,3,4] could be made into six possible three-digit numbers, namely 234, 243, 324, 342, 423, and 432. I knew there had to be 6 = 3×2×1 possible rearrangements, since there were three choices for the first digit, two choices for the second digit, and one choice for the final digit.

Similarly, [1,4,6] could be made into six possible three-digit numbers by the exact same argument: 146, 164, 416, 461, 614, and 641.

Finally, [2,2,6] could only be made into three possible three-digit numbers since the digit 2 appeared twice. The possible options were 226, 262, and 622.

The correct answer was 6+6+3 = 15, and I was all done with some time to spare! All I had to do was fill in the right bubble.

But 15 was not listed as any of the multiple-choice answers. What happened? Did the contest organizers screw up? Where was 15?

Or did I make a mistake? Did I miss a case?

Did I forget about the O-ring?

I started to get anxious. What did I forget? What else was there besides 2×3×4, 2×2×6, and 1×4×6?

Did I read the question wrong?

Each of the integers 226 and 318 have digits whose product is 24. How many three-digit positive integers have digits whose product is 24?

No, I definitely read the problem right. So what happened? My mind started to race.

Relax, Bethany. Calm down.

I took a deep breath and closed my eyes for twenty seconds.

When I opened my eyes, I re-read the question. *Each of the integers 226 and 318 . . .*

And then I saw it.

Right in the statement of the problem, the number 318 was written. I had forgotten the case 3×1×8, the other instance of three digits multiplying to 24.

From the missing case [1,3,8], I quickly determined the six additional three-digit numbers whose digits multiplied to 24, namely 138, 183, 318, 381, 813, and 831.

So my answer wasn't 15, but 15+6.

Did 21 appear as one of the five possible multiple-choice options? To my relief, it did.

I bubbled in the right letter and dropped my pencil on the table. I closed my eyes and wanted to scream.

Yes, I got them all!

I knew I had nailed everything in Part A, and so I didn't bother to re-check those ten questions. I skimmed through the ten questions in Part B, making sure I didn't make any careless computation errors.

11:55 a.m.

I went through the five questions in Part C, double-checking and triple-checking that I didn't do anything incorrectly. Everything was fine. I glanced up at the clock, seeing that the minute hand was now touching the hour hand.

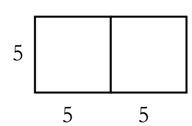
"Thirty seconds," said Miss Carvery.

My heart was pounding. I was going to get a perfect score.

I opened the contest to the first page, where all the Part A questions were. My face froze when I saw Question #10 and the diagram that I had marked on the contest booklet.

10. Two squares, each with an area of $25\ cm^2$ are placed side by side to form a rectangle. What is the perimeter of this rectangle?

(A) 30 cm (B) 25 cm (C) 50 cm (D) 20 cm (E) 15 cm



P = 10 + 5 = 15

The length was 10 and the width was 5. Since the perimeter represented the sum of the lengths of all *four* sides, the perimeter wasn't 10+5=15, but 10+5+10+5=30. Oh no!

Quickly grabbing my eraser, I found Question #10 on the answer sheet, erased my incorrect answer and bubbled in the correct letter.

"Okay, the contest is done. Stop writing!" said Miss Carvery.

I let out a deep breath, and dropped my pencil on the table.

"Please leave your answer sheet on the table," said Miss Carvery. "Congratulations to you all. The pizza will be arriving in a few minutes."

My heart continued to pound. I had trouble breathing.

I messed up Question #10 but was just barely able to fix it. What other careless mistakes did I make?

I didn't bother to check my answers to the Part A questions since I was sure I got them all, but did I? If I screwed up Question #10, what else did I get wrong?

M-A-R-R-O-O-N.

I felt like throwing up.

Miss Carvery walked by my desk to pick up my answer sheet.

"Bethany, are you okay?"

I shook my head.

"Do you need some fresh air?"

I nodded. I stood up and ran to the washroom. I hurried into a stall, locked the door, knelt down next to the toilet bowl, and lifted the seat.

With a great heave, I barfed up my entire breakfast. There were pieces of dried toast and peanut butter everywhere. I started to breathe a bit slower and felt my stomach calm down.

I sat down for a couple of minutes on the cold tiled floor, and leaned over to flush the toilet.

"Bethany."

It was Miss Carvery. She knocked on the stall.

"Could you open the door please?"

I turned the lock and Miss Carvery came in. The vice-principal sat down next to me and put her hand on my shoulder. She gave me a bottle of water, which I gladly accepted.

"Do you want me to stay with you?"

I nodded. Miss Carvery told me to take deep breaths, which I did, many times. I started to relax.

We eventually made it back to the classroom. Walking next to Miss Carvery, I noticed that all the students were sitting around the tables in the back, comparing their answers while eating pizza.

Gillian saw us and pounced immediately.

"Bethany cheated, Miss Carvery."

What?

"That's a serious accusation," said Miss Carvery. "What evidence do you have?"

Gillian marched to my desk and held up the sheet of paper with **s t o p** written in large block letters.

"I saw her take this page and hold it up to the light, seeing what the reflection would look like. That's how she solved Question #20. That's cheating."

I was speechless.

"Gillian, take it easy. Bethany's not feeling well."

"But Miss Carvery, it's not fair! I did the questions properly. If Bethany gets the highest mark in the school, then it's because she cheated. You have to disqualify her, Miss Carvery!"

"Gillian, calm down. I'll take care of this."

Miss Carvery turned to me and motioned towards the door. "Bethany, come to my office. I want to talk with you privately."

Gillian looked smug and glared at me, before walking back to join Vanessa.

I followed Miss Carvery to her office, the one with VICE-PRINCIPAL written in big bold letters. I was scared and nervous for what was about to unfold.

Before Miss Carvery closed her door, she motioned to one of the administrative staff and gave him a ten dollar bill. She whispered something that I couldn't hear.

Miss Carvery turned to face me. I needed to set the record straight.

"I promise I didn't cheat."

"I believe you, Bethany. Please, have a seat."

"Yes, I did hold that page up to the light, but there's no rule saying I couldn't. Mr. Collins told me I could use props if I wanted, if it would help me solve the problem faster. He said it wasn't cheating – he called it 'creative problem-solving'."

Miss Carvery laughed. "That sounds like exactly something Taylor Collins would say."

"You know him, right?" I asked.

"Indeed. I'm a graduate of Sydney High School. Mr. Collins was my teacher too."

"So you don't think I cheated?"

"Of course not. Gillian is just overreacting, but let's just keep that between us. Don't worry about her."

"She's so mean."

"Gillian can say mean things, but you don't know her situation. Let me just say that Gillian has a lot of pressure to be the top student in the class. You threaten her, Bethany. You're the only person who can challenge her. You're the only person who is just as bright and clever as she is. If Gillian talks down to you, it's not because she's mean; it's because she's intimidated by you."

I heard a knock on the door. Miss Carvery opened it, and returned holding a tray with two sandwiches and two small cartons of milk. She handed one of the sandwiches to me.

"After what happened, I thought you would prefer eating this, rather than pepperoni and bacon pizza."

I smiled in appreciation, as I took off the plastic wrapping and took a bite.

"Gillian is intimidated by me?" I asked, my mouth full of tuna and celery.

"Yes. And Gillian knows that you live in a great home. Perhaps that also makes her jealous."

I was confused. "But Gillian lives in the biggest house in Cape Breton. She has a pool!"

"A house is different from a home, Bethany. You have something that she doesn't."

"What's that?"

Miss Carvery paused and took a bite of her sandwich. She hesitated.

"Your mother loves you and is supportive of everything you do."

"Thank you," I replied, realizing the implication of Miss Carvery's words.

Even though Mom and I argued sometimes, we were really close. We watched movies together and played Scrabble in the evening. She continued in the government job that she hated because it provided a secure and comfortable life for me. She taught figure skating to Ella Collins so that her grandfather could be my coach. Mom was awesome.

"I wish I could do something for Mom."

"You already are," said Miss Carvery. "You're living your life. And I'm sure she's secretly glad that your passion is math, not figure skating."

"Mom doesn't like to talk about figure skating," I said, tensing up. "She always changes the subject. We can talk about anything but that's the one subject that's off-limits."

Miss Carvery nodded. "Given what happened, it's natural Lucy would feel that way."

My heart stared to beat rapidly. I needed Miss Carvery to answer a question that I'd been carrying with me since the day of the Spelling Bee.

"Why were you shocked when you heard Mom was coaching Ella?"

Miss Carvery paused. She thought for a long time before answering.

"Because of what happened to your mother at the nationals. We were all devastated when she didn't make the team."

"What team?" I asked, confused.

She stared at me. Seeing my blank look, she looked away.

"It was the Olympic team, wasn't it?" I whispered.

Miss Carvery nodded.

Everything that Mom said, and didn't say, suddenly made sense.

I finally understood why Mom didn't want me to pursue the Math Olympiad, why she always changed the subject when it came to figure skating, and why she was in so much pain to talk about her past.

Oh my God, how did I not realize this earlier?

"Bethany, your face is all blue!" said Miss Carvery.

My mind flashed back to the conversations we had last summer, about the dangers of pursuing unrealistic goals and the risks of putting all of our eggs in one basket. I remember Mom crying as she talked about her shattered dream. I had no idea what that dream was . . . until now.

How could I have been so blind?

"Bethany, talk to me. What's wrong?"

I couldn't answer. My face, streaked with tears, felt the burden of Mom's pain over all these years. As she saw me getting excited about math and maybe one day becoming an Olympian like Rachel Mullen, she must have been constantly reminded of the memories of her own childhood, and all the sacrifices she was forced to make as she sought to become an Olympian herself.

I unfairly assumed that she was a bad mother who tried to discourage my dreams, instead of a loving mother who tried to protect me from pain.

How could I have been this selfish?

Miss Carvery sat down beside and held my hand. She handed me a tissue, and wrapped me in a hug.

I sobbed on her shoulder and couldn't reply. She just held me, seemingly not bothered by the large wet patch forming on her suit jacket. After a few minutes, she looked gently into my eyes.

"You never knew, did you?"

The Canadian Mathematical Olympiad, Problem #1

Determine the value of

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{3/1000}}{9^{3/1000}+3}+\cdots+\frac{9^{998/1000}}{9^{998/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}$$

Solution to Problem #1

As I close my eyes, the memories come back in a flash, one after the other.

The staircase in Mrs. Ridley's class, the visit to the hospital that brought Mr. Collins into my life, learning how to cross-train my mind, finding patterns by simplifying hard problems into easier components, the euphoria of achieving a perfect score on my very first math contest, and learning Mom's secret.

All of this happened years ago, before I even entered high school. But the experiences were so memorable that I can still remember every detail.

Years later, I find myself in this boardroom, writing the Canadian Mathematical Olympiad for a spot on Canada's IMO team. And I'm stunned that the key idea for the first problem is *The Staircase*. I re-read Problem #1.

Determine the value of:

$$\frac{9^{1/1000}}{9^{1/1000}+3}+\frac{9^{2/1000}}{9^{2/1000}+3}+\frac{9^{3/1000}}{9^{3/1000}+3}+\cdots+\frac{9^{998/1000}}{9^{998/1000}+3}+\frac{9^{999/1000}}{9^{999/1000}+3}$$

In my simplified problem with three terms instead of 999, I want to determine the value of $\frac{9^{1/4}}{9^{1/4}+3}+\frac{9^{2/4}}{9^{2/4}+3}+\frac{9^{3/4}}{9^{3/4}+3}$. I've already determined that the middle term is equal to one-half.

$$\frac{9^{2/4}}{9^{2/4}+3}=\frac{9^{1/2}}{9^{1/2}+3}=\frac{\sqrt{9}}{\sqrt{9}+3}=\frac{3}{3+3}=\frac{3}{6}=\frac{1}{2}$$

I look down at my notes for calculating the other two terms, applying various properties of exponents.

The first term is $\frac{9^{1/4}}{9^{1/4}+3}=\frac{(3^2)^{1/4}}{(3^2)^{1/4}+3}=\frac{3^{1/2}}{3^{1/2}+3}=\frac{\sqrt{3}}{\sqrt{3}+3}=\frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}\times(\sqrt{3}+3)}=\frac{3}{3+3\sqrt{3}}$.

The last term is $\frac{9^{3/4}}{9^{3/4}+3}=\frac{(3^2)^{3/4}}{(3^2)^{3/4}+3}=\frac{3^{3/2}}{3^{3/2}+3}=\frac{3\sqrt{3}}{3\sqrt{3}+3}=\frac{3\sqrt{3}}{3+3\sqrt{3}}$.

Instead of determining the value of each term separately before computing its sum, I realize I can just add the two expressions directly, producing a fraction with the same numerator and denominator.

$$\frac{9^{1/4}}{9^{1/4}+3} + \frac{9^{3/4}}{9^{3/4}+3} = \frac{3}{3+3\sqrt{3}} + \frac{3\sqrt{3}}{3+3\sqrt{3}} = \frac{3+3\sqrt{3}}{3+3\sqrt{3}} = \mathbf{1}$$

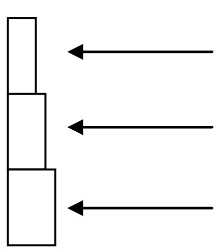
The first and last terms, when paired, add up to one. When I see that this total is just one, I have a hunch that the same property is true in the much-harder Olympiad problem.

Sure enough, it is. I'm able to show that when we pair the terms from the two endpoints, its sum is always one. For example,

$$\frac{9^{1/1000}}{9^{1/1000}+3} + \frac{9^{999/1000}}{9^{999/1000}+3} = \mathbf{1}, \quad \frac{9^{2/1000}}{9^{2/1000}+3} + \frac{9^{998/1000}}{9^{998/1000}+3} = \mathbf{1}$$

Armed with this insight, there is a simple way to determine the sum, using the staircase picture. Just to make sure I have all the details correct, I first apply it to my simplified problem with three terms. I draw three rectangles, each with height 1. I make the width in the first rectangle $\frac{9^{1/4}}{9^{1/4}+3}$ the second rectangle $\frac{9^{2/4}}{9^{2/4}+3}$ and the third rectangle $\frac{9^{3/4}}{9^{3/4}+3}$.

I attach the three rectangles together to form a staircase. Since each rectangle has height 1, the staircase has total area $\frac{9^{1/4}}{9^{1/4}+3} + \frac{9^{2/4}}{9^{2/4}+3} + \frac{9^{3/4}}{9^{3/4}+3}$.

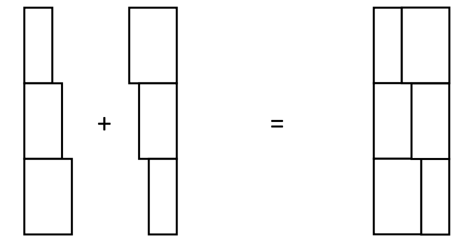


This step has area $1 \times \frac{9^{1/4}}{9^{1/4}+3} = \frac{9^{1/4}}{9^{1/4}+3}$

This step has area $1 \times \frac{9^{2/4}}{9^{2/4}+3} = \frac{9^{2/4}}{9^{2/4}+3}$

This step has area $1 \times \frac{9^{3/4}}{9^{3/4}+3} = \frac{9^{3/4}}{9^{3/4}+3}$

Like I did back in Grade 5, I'm able to represent the desired sum visually. But this time, instead of counting the number of squares in the staircase, I want to find its combined area. To do this, I flip the staircase diagram, and paste it to the original figure.



So the two staircases become one simple rectangle, since the width of each "block" is guaranteed to be 1 by how I've done my pairing. Therefore, the area of the two-staircase figure is just 3 times 1, the height of the rectangle multiplied by its width. And so the area of a single staircase is 3/2, or half of 3.

Without doing any calculations, I know that the answer to the Olympiad Problem is 999/2, by the exact same argument. I start writing my solution. To ensure I finish as quickly as possible, I decide to use "function notation", $f(x)$, and "sigma notation", Σ, a beautiful way to use mathematical symbols without losing any meaning or rigour. I complete the solution and quickly re-read what I've written.

It's perfect.

One done, four to go.

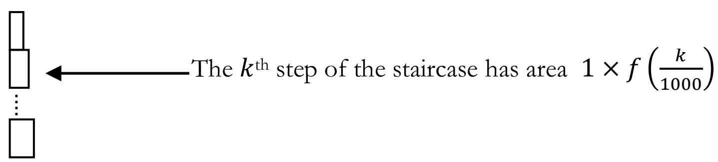
Problem Number: 1
Contestant Name: Bethany MacDonald

Let $S = \frac{9^{1/1000}}{9^{1/1000}+3} + \frac{9^{2/1000}}{9^{2/1000}+3} + \cdots + \frac{9^{999/1000}}{9^{999/1000}+3}$. We claim that $S = \frac{999}{2}$.

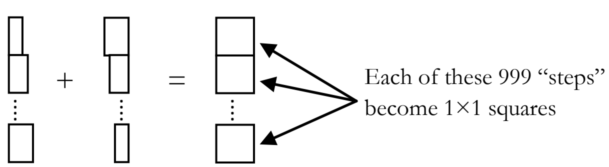
Let $f(x) = \frac{9^x}{9^x+3}$. We first show that $f(x) + f(1-x) = 1$ for any real value of x. This is true because $f(x) + f(1-x)$ equals

$$\frac{9^x}{9^x+3} + \frac{9^{1-x}}{9^{1-x}+3} = \frac{9^x(9^{1-x}+3)+9^{1-x}(9^x+3)}{(9^x+3)(9^{1-x}+3)} = \frac{9+3\cdot 9^x+9+3\cdot 9^{1-x}}{9+3\cdot 9^x+9+3\cdot 9^{1-x}} = 1$$

Now consider a "staircase" diagram with 999 steps, each of height 1, where the k^{th} step has length $f\left(\frac{k}{1000}\right)$. Then the area of this staircase is $\sum_{k=1}^{999} f\left(\frac{k}{1000}\right)$, which is equal to S.



Now invert a copy of the staircase and attach it to the original so that the k^{th} step in the first staircase is joined to the $(1000-k)^{\text{th}}$ step in the second staircase, as shown in the diagram below.



The area of this new figure is $2S$, since it is just the piecing together of two staircases with area S. By this construction, the k^{th} step in the new figure has length $f\left(\frac{k}{1000}\right) + f\left(\frac{1000-k}{1000}\right) = f\left(\frac{k}{1000}\right) + f\left(1 - \frac{k}{1000}\right) = 1$.

It follows that each of the 999 "steps" in the new figure has area 1, since it is just a square with height 1 and length 1. Therefore, we have $2S = \sum_{k=1}^{999} f\left(\frac{k}{1000}\right) + f\left(\frac{1000-k}{1000}\right) = 999 \times 1 = 999$, implying that $S = \frac{999}{2}$.

Our proof is complete.

The Canadian Mathematical Olympiad, Problem #2

Find all real solutions to the following system of equations.

$$\begin{cases} \dfrac{4x^2}{1+4x^2} = y \\[2ex] \dfrac{4y^2}{1+4y^2} = z \\[2ex] \dfrac{4z^2}{1+4z^2} = x \end{cases}$$